Comparative analysis of mathematics teachers' content knowledge in USA and Russia through the lens of TIMSS results

Mourat Tchoshanov[1], Maria Cruz Quinones[2], Kadriya Shakirova[3], Elena Ibragimova[4] and Liliana Shakirova[5]

[1]*University of Texas at El Paso, USA; mouratt@utep.edu*

ORCID 0000-0002-2852-4311

[2]*Universidad Autónoma de Cuidad Juárez, Mexico; titacq@hotmail.com*

ORCID 0000-0003-2107-6568

[3]*Kazan Federal University, Russian Federation; shakirova_ka@mail.ru*

ORCID 0000-0002-4433-6740

[4]*Kazan Federal University, Russian Federation; esandakova@mail.ru*

ORCID 0000-0001-9271-0170

[5]*Kazan Federal University, Russian Federation; Liliana.Shakirova@kpfu.ru*

ORCID 0000-0001-5758-4076

Abstract

The quantitative study focused on comparative analysis of middle school mathematics teachers' content knowledge in two countries. The sample consisted of lower secondary mathematics teachers from the US (grades 6-9, N=102) and Russia (grades 5-9, N=97). The instrument was designed to assess teacher content knowledge based on the cognitive domains of knowing, applying, and reasoning, as well as addressing the lower secondary mathematics topics of Number, Algebra, Geometry, Data and Chance. The results suggest that there are significant differences in teacher knowledge between the countries in content as well as in cognitive domains. The study results may inform the field on priorities placed on lower secondary mathematics teachers' knowledge in USA and Russia.

Keywords: comparative studies, teacher knowledge, lower secondary mathematics.

Сравнительный анализ предметных знаний учителей математики в США и России через призму результатов TIMSS

Мурат Чошанов[1], Maria Cruz Quinones[2], Кадрия Б. Шакирова[3], Елена Н. Ибрагимова[4] и Лилиана Р. Шакирова[5]

[1]*Техасский университет в Эль-Пасо, США; mouratt@utep.edu*

ORCID 0000-0002-2852-4311

[2]*Университет Автонома-де-Сьюдад-Хуарес, Мексика; titacq@hotmail.com*

ORCID 0000-0003-2107-6568

[3]*Казанский федеральный университет, Казань, Россия; shakirova_ka@mail.ru*

ORCID 0000-0002-4433-6740

[4]*Казанский федеральный университет, Казань, Россия; esandakova@mail.ru*

ORCID 0000-0001-9271-0170

[5]*Казанский федеральный университет, Казань, Россия; Liliana.Shakirova@kpfu.ru*

ORCID 0000-0001-5758-4076

Аннотация
Количественное исследование сосредоточено на сравнительном анализе предметных знаний учителей математики в двух странах. Выборка состоит из учителей математики среднего звена школы в США (6-9 классы, N = 102) и России (5-9 классы, N = 97). С этой целью был разработан тест для оценки предметных знаний учителя на основе когнитивных уровней (знания, применения, и рассуждения), а также содержательных тем курса математики среднего звена школы: арифметики, алгебры, геометрии, вероятности и статистики. Полученные результаты свидетельствуют о том, что существуют статистически значимые различия в знаниях учителей между странами в содержании, а также по когнитивным уровням. Результаты исследования могут информировать теорию и практику сравнительных исследований о приоритетах в подготовке учителей математики среднего звена школы в США и России.
Ключевые слова: сравнительные исследования, знания учителя, среднее звено школы.

Introduction
The motivation for the study is based on the 8th-grade mathematics portion of the TIMSS-2011 results (Mullis et al. 2012). We identified two countries ranked closely to each other: Russia – in the 6th position and the USA – in the 9th position. At the same time, a difference in the US and Russian students' scores was revealing: the average score of Russian students in the content domain was 539 and of the US students 509, with Russian students gaining higher scores on Number (534 vs. 514), Algebra (556 vs. 512) and Geometry (533 vs. 485) whereas US students outscored Russian students in the domain of Data and Chance (527 vs. 511). Russian students also outperformed the US students in each cognitive domain: Knowing (548 vs. 519) Applying (538 vs. 503), and Reasoning (531 vs. 503). These data triggered a question on whether the difference in student performance could be explained by the difference in teacher performance. Therefore we selected the same two countries (e.g., US and Russia) and tested lower secondary mathematics teachers' knowledge by content and cognitive domains using a similar instrument.

Cross-national studies of teacher knowledge
Before we start the theoretical part, we state the research question that guided our cross-national study: to what extend the US and Russian lower secondary mathematics teachers' knowledge differ by content and cognitive domains?

Conducting cross-national studies allow comparing, sharing, and learning about issues in an international context (Robitaille & Travers, 1992). Cross-national studies also help researchers understand in a more explicit way about their own context, teaching practice, knowledge, and get insights of better choices in constructing the teaching and learning process (Stigler and Perry, 1988). During the last decade, the number of cross-national studies on teacher education is increasing in order to understand differences in student performance on international tests such as TIMSS, PISA (Wang & Lin, 2005). Scholars have addressed these differences through variety of quantitative, qualitative and mixed methods studies focusing on characteristics such as teachers' perceptions of effective mathematics teaching (Cai, Ding, & Wang, 2013; Hemmi & Ryve, 2015), attitudes and beliefs of mathematics pre-service teachers (Wagner, Lee, & Ozgun-Koca, 1999), teacher knowledge (TEDS-M, 2011; Tchoshanov et al., 2015), among others.

Most of the prior studies focused on the affective domain of mathematics instruction. Cai, Ding, and Wang (2013) conducted a cross-national study to examine the US and Chinese in-service teachers' (n=36) view about the meaning of instructional coherence. The study found that instructional coherence is highly related to discourse. In regards to teacher view of effective mathematics teaching, Hemmi and Ryve (2014) conducted a cross-national study of teacher-educators' perception of effective mathematics teaching in Sweden (n=8) and Finland (n=5). The study found that Swedish teacher educators conceptualize effective teaching as interactions with individual children, building on students´ ideas and using mathematics from emerging situations. Finnish teacher educators consider effective teaching in providing a clear presentation of mathematics, routines, and homework.

Other studies focused on the cognitive domain of mathematics instruction. For example, Andrews (2008) compared middle school teachers' (n=16) conceptualization and presentation of mathematics in four countries: England, Belgium, Hungary and Spain. The Flemish teachers present a moderate cognitive complexity supported by a moderately high didactic coherence while English teachers present a low cognitive complexity and barely moderate didactic coherence. The Hungarian teachers presented a high cognitive complexity supported by high levels of didactic coherence in contrast with the Spanish teachers comprising low cognitive complexity allied to low didactic coherence.

Few cross-national studies focused on teacher knowledge. A large-scale study conducted by the University of Michigan examined the mathematical content and pedagogical content knowledge of pre-service teachers from 17 countries including USA and Russia (Tatto & Senk, 2011; Tatto, 2013). The development of the items for the TEDS-M study was informed by MT21 knowing mathematics for teaching (Ferrini-Mundy, Floden, McCrory, Burril, & Sandow, 2005) and learning mathematics for teaching frameworks (Hill, Ball, and Schilling, 2008). The nature of mathematics teacher knowledge, conceptual representation, and curriculum materials were examined by Ma (1999) to explain differences in students´ performance in the U.S. and China. An, Kulm, and Wu (2004) studied the PCK of middle school teachers (n=61) in the U.S. and China. They found that mathematical PCK differs since Chinese teachers emphasize developing procedural and conceptual knowledge through traditional teaching practices while their counterparts in the U.S. focus on promoting creativity and inquiry through activities designed to develop student´s understanding of mathematical concepts. Sorto et al. (2009) administered surveys that measured teachers' content knowledge (n=385) in Costa Rica and Panama and found that teachers in both countries focus more on knowing rules and procedures than on making connections and reasoning.

The literature review indicates that there is a need to conduct cross-national studies with a particular focus on in-service teachers' knowledge and its potential impact on students' learning and achievement in mathematics.

Teacher knowledge

The field of mathematics education is expanding its knowledge-base in understanding the role of teacher characteristics in student learning and achievement. The major shift in the field had happened with Shulman's (1986) work on teacher knowledge that proposed an alternative approach to the educational production function perspective (e.g., Hanushek, 1981, Monk & Rice, 1994), which was concerned with examining proxies of teacher knowledge such as coursework/certification and its impact on student achievement (Charalambous & Pitta-Pantazi, 2016). Research on teacher knowledge initiated by work of Shulman (1986) has focused on teacher knowledge as a major predictor of student learning and achievement. In the last decade, the field benefited from numerous studies (Hill, Shilling, & Ball, 2004; Hill, Ball, & Schilling, 2008; Rowland, Huckstep, & Thwaites, 2005; Davis & Simmt, 2006; Baumert et al., 2010) that substantially advanced the conceptualization of teacher knowledge.

Capitalizing on Shulman's (1986) work, scholars examined different categories of teacher knowledge. Content or subject-matter knowledge and pedagogical content knowledge are the most important categories of teacher knowledge. Bransford, Brown, and Cocking (2000) state that content knowledge requires "a deep foundation of factual knowledge, understanding of the facts and ideas in the context of a conceptual framework, and organization of the knowledge in ways that facilitate retrieval and application" (p. 16). Hill, Ball, and Schilling (2008) consider a special kind of teacher knowledge that combines content and pedagogical content knowledge – mathematical knowledge for teaching. It is knowledge "that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (p. 378).

Some scholars (e.g., Chapman, 2013; Izsak, Jacobson, & de Araujo, 2012) examined different facets of teacher knowledge without explicitly emphasizing its connection to student learning. Other scholars stressed the importance of the kind of knowledge a teacher possesses because it impacts his/her teaching (Steinberg, Haymore, and Marks, 1985). Another line of research (e.g., Hill, Rowan, & Ball, 2005; Baumert et al, 2010; Tchoshanov, 2011) specifically targets the effects of different types of teachers' knowledge on student achievement. There is a need in the field for extending the latter line of research to the level of cross-national studies on teacher knowledge and its connection to student achievement.

Recently, scholars have advanced the field by examining teacher knowledge in variety of domains including Number Sense (Ma, 1999; Izsac, 2008), Algebra (Bair & Rich, 2011; McCrory et al., 2012), Geometry and Measurement (Murphy, 2012; Nason, Chalmers, & Yeh, 2012), and Statistics (Groth & Bergner, 2006). However, the field lacks cross-national research that provides a comprehensive analysis of the various facets of teacher knowledge (including content and cognitive domains) and its connection to student performance.

Methodology

The proposed study is based on the assessment framework used by TIMSS (Mullis et al. 2012). In this section, we will describe the study participants, the instrument as well as data collection and data analysis procedures.

Participants

The convenience sampling technique was employed to select study participants. The sample consisted of lower secondary mathematics teachers from the US (grades 6-9, N=102) and Russia (grades 5-9, N=97). The US teacher-participants were selected from urban public middle schools in the Southwestern part of the country. Teacher sample demographic information was self-reported by participating teachers. In terms of gender distribution, 55% of teacher participants were females and 45% – males. Most of the US participants (64%) had 1-5 years of teaching experience. Additionally, 62% of the teacher sample received their teaching certificate through traditional teacher preparation programs and 38% of participating teachers were certified through alternative programs. The Russian teacher-participants were selected from urban public secondary schools in the Volga region. Russian participating teachers had attained a secondary mathematics teacher preparation Specialist's degree[1], which allowed them to teach in secondary schools (grades 5-11). The majority of participating teachers were females (89%). The sample was composed of 78% of teachers who have more than 10 years of teaching experience.

[1] In Russia, the secondary school consists of lower and upper levels: the lower secondary school includes grades from 5 to 9, and grades 10-11 are part of the upper secondary school. In USA, the secondary school consists of middle school, which might include grades 6-9 (depends on a particular state) and high school (grades 9-12). Curriculum wise, there might be an overlap between middle and high school at grade 9 which focuses mainly on topics addressing Algebra-1.

Instrument

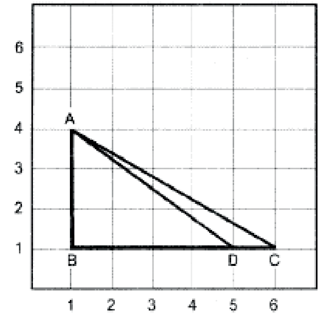
The instrument used in this study was the Teacher Content Knowledge Survey (TCKS) which was specifically developed using TIMSS framework (Mullis et al. 2012). The survey was designed to measure teacher content knowledge based on four content domains (Number, Algebra, Geometry, and Data and Chance) and three cognitive domains: Knowing, Applying, and Reasoning. The TCKS instrument included 33 multiple-choice items topics addressing main objectives of lower secondary mathematics curriculum. Examples of the TCKS items are presented below.

1. From the following set, how many numbers are irrational numbers?

$$S = \{\sqrt{4}, \sqrt{6}, \sqrt{8}\}$$

A. 0
B. 1
C. 2
D. 3

Use the following diagram to answer questions (2) and (3).



2. Which of the following measurements associated with the triangles in the diagram above can be expressed as a non-terminating, non-repeating decimal?

A. AD
B. AC
C. BC
D. DC

3. If point C moves continuously along the ray $\overrightarrow{BD}$, the length of segment AC takes on values that are:

A. Mostly rational
B. Mostly irrational
C. Equal amount of rational and irrational
D. Insufficient information provided.

The survey was designed by interdisciplinary faculty representing university, community college, and local schools. The item development process was guided by a list of descriptors for each cognitive domain. The list for the Knowing domain included, but was not limited, to the following descriptors: recognize basic terminology and notation, recall facts, state definitions, name properties and rules, do computations, make observations, conduct measurements, simplify and evaluate numerical expressions. The list for Applying domain consisted of the following descriptors: select and use appropriate representation, translate between representations, transform within the same representation, transfer knowledge to a new situation, connect two or more concepts, explain and justify solutions, communicate mathematical ideas, explain findings and results from analysis of data. And, the list for the Reasoning domain included, but was not limited, to the following descriptors: generalize patterns, formulate mathematical problems, generate mathematical statements, derive mathematical formulas, make predictions and hypothesize, design mathematical models, extrapolate findings from data analysis, test conjectures, prove statements and theorems, solve non-routine problems. As mentioned by one of our esteemed reviewers there might be a slight overlap between descriptors for different cognitive domains that is not critically affecting the distinct nature of each domain: Knowing is declarative, Applying is procedural, and Reasoning is conceptual.

The content validity was established through construction of a specification table to guide the process of the instrument development. Another table, an item analysis table was constructed to address a construct validity of the TCKS. The instrument was piloted with the follow up revision of the items to produce the final version of the instrument. The reliability of the TKCS was checked using Cronbach alpha coefficient technique (Cronbach, 1951). "The value of the coefficient of .839 suggests that the items comprising the TCKS are internally consistent" (Tchoshanov, 2011).

Data Collection

The measurement of teachers' knowledge was conducted using the TCKS instrument. Each teacher was given 90 min to complete the survey. Along with teachers' scores on the TCKS, teachers' demographic information such as gender and ethnicity, years of teaching experiences, as well as other proxies for teacher content knowledge (i.e., mathematics coursework) were also collected.

Data Analysis

In correspondence with the research question, data analysis was performed using non-parametric techniques (chi-square test of goodness of fit). This statistic was selected to measure the variance between independent groups of the same (not normal) distribution with arbitrary sample sizes of each group. The selection of this test was also based on the ordinal (ranked) nature of data for content and cognitive domains of teacher knowledge and student performance.

Results

In this section, we first analyze teacher knowledge data by content domain, then we analyze teacher data by cognitive domain, and finally we analyze parallels between student and teacher performance within and between countries.

The results reported on teacher content knowledge show that the US teachers' highest mean score was obtained on Number domain – 623 and lowest on Geometry domain – 514 (see Table 1).

Table 1. *US teachers´ means scores by content domain*

Content Domain	Mean	SE	SD	Conf. level (95%)
Number	623	20.3129	205.1512	40.296
Algebra	563	23.2356	234.6679	46.093
Geometry	514	25.4349	256.8802	50.456
Data and Chance	593	20.9738	211.8252	41.606

Russian teachers' highest mean score was obtained on Algebra domain – 728 and lowest on Data and Chance domain – 387 (see Table 2).

Table 2. *Russian teachers´ means scores by content domain*

Content Domain	Mean	SE	SD	Conf. Level (95%)
Number	656	106.5819	319.7456	23.873
Algebra	728	82.8841	248.6523	30.648
Geometry	586	72.7004	218.1013	45.505
Data and Chance	387	125.0891	306.4044	35.844

Moreover, we found that the US teachers' highest mean score was obtained, as expected, on Knowing domain – 734 and lowest on Reasoning domain – 495 (see Table 3).

Table 3. *US teachers´ means scores by cognitive domain*

Cognitive Domain	Mean	SE	SD	Conf. level (95%)
Knowing	734	19.7673	197.6733	39.2226
Applying	505	20.7101	207.1015	41.0934
Reasoning	495	23.8130	238.1303	47.2502

Russian teachers' highest mean score was obtained, as expected, on Knowing domain – 760 and lowest, unexpectedly, on Applying domain – 504 (see Table 4).

Table 4. *Russian teachers´ means scores by cognitive domain*

Cognitive Domain	Mean	SE	SD	Conf. level (95%)
Knowing	760	14.2486	135.1745	28.3117
Applying	504	12.7961	121.3950	25.4257
Reasoning	593	17.7406	168.3028	35.2503

Moreover, we identified that there is no significant difference between Russian and US teachers' knowledge on Number and Geometry domains (*Chi-square 0.347 $p>.05$ and Chi-square 1.293 $p>.05$*) (see Table 5).

Table 5. *Russian and US teachers' knowledge by content domain*

Content Domain	Number	Algebra	Geometry	Data and Chance
Russia	656	728	586	387
USA	623	563	514	593
Chi-square	0.347	6.311*	1.293	8.003**
p-value	0.5558	0.0119	0.2555	0.0047

However, there is a statistically significant difference between Russian and US teachers' knowledge on Algebra domain (in favor of Russian teachers; *Chi-square 6.311 $p<.05$*) and Data and Chance domain (in favor of US teachers; *Chi-square 8.003 $p<.05$*) (see Table 7). This finding closely parallels the US and Russian *students' performance* on

TIMSS on Algebra domain (in favor of Russian students) and Data and Chance domain (in favor of US students).

Also, this study reported that there is no significant difference between Russian and US teachers' knowledge on Knowing and Applying cognitive domains (*Chi-square 1.707 p>.05 and Chi-square 0.008 p>.05*) whereas there is a statistically significant difference on Reasoning domain (in favor of Russian teachers; *Chi-square 19.117 p<.05*) (see Table 6).

Table 6. *Russian and US teachers' knowledge by cognitive domain*

Cognitive Domain	Knowing	Applying	Reasoning
Russia	760	504	593
USA	734	505	495
Chi-square	1.707	0.008	19.117**
p-value	0.1914	0.9287	0

This finding parallels the US and Russian students' performance on TIMSS' cognitive domain.

Furthermore, cross-national curriculum analysis shows that Russian teachers have more extensive content preparation compare to their American counterparts. A number of contact hours for mathematical content knowledge, as well as pedagogical content knowledge and specialized mathematics knowledge offered at selected teacher preparation programs (e.g., the University of Texas at El Paso, USA and Kazan Federal University, Russia) in two countries, are presented in table 7.

Table 7. *Contact hours in Mathematics related disciplines in teacher education programs in Russia and United States*

Country	Mathematics Content Knowledge (Academic Mathematics)	Pedagogical Content Knowledge (Mathematics Pedagogy)	Specialized Mathematics Knowledge (School Mathematics)
Russia	1857	278	380
United States	442	72	87

Numbers depicted in the table are compatible with the findings of the TEDS-M study (Tatto & Senk, 2011).

Discussion and Conclusion

This study confirms the differences between Russian and the U.S. lower secondary in-service teachers' knowledge in the content domain as it was reported by the TEDS-M study that was focused on pre-service teachers (Tatto & Senk, 2011). At the same time, this study expands the examination of in-service teachers' knowledge to the cognitive domain.

Teacher preparation could be considered as the main factor contributing to the differences between Russian and US teachers' knowledge. Overall, there is a tangible difference in secondary teacher preparation curriculum between the two countries: in average, Russia offers about 240 credit hours in teacher preparation programs compare to 120 credits in the USA.

Close examination of secondary teacher preparation curriculum in Russia shows more emphasis placed on an analytic and algebraic component of mathematics and less emphasis on statistic and probability component compare to the US curriculum.

Moreover, item analysis of standardized tests for the lower secondary schools in USA and Russia revealed the difference in selection and composition of algebra problems as well as problems related to data and chance in the test: while in Russia more emphasis is placed on algebraic problems and less emphasis on data and chance problems, in the USA – the emphasis is equally distributed among algebraic problems and data and chance problems. We observed another noticeable difference in the role of proof in the academic mathematics component of the teacher preparation program which could explain the difference in the reasoning domain of the teacher knowledge: Russian curriculum places a heavy emphasis on proof across the mathematics coursework including school mathematics whereas the US curriculum uses proof in selected mathematics courses primarily in academic mathematics coursework.

We are cognizant of the limitations concerning the convenient sampling technique that influences generalizability of the study results. Moreover, there is no cluster matching between teachers participating in the study and students tested in TIMSS. However, the study main results suggest that student performance on international tests could be explained by teacher knowledge. The study also presents opportunities for comparing, sharing, and learning about issues in cross-national context in US and Russian teacher education, training, and development. Moreover, the reported cross-national study on teacher knowledge may inform the field on priorities placed on lower secondary mathematics teachers' knowledge in USA and Russia by content and cognitive domains.

Statements on open data, ethics and conflict of interest

The data used in the report could be accessed through the direct inquiry sent to one of the co-authors of the paper with a clear rational for the purpose of using the data. A secondary data (TIMSS results) could be accessed via the following site: www.timss.org.

The ethical guidelines, under which the reported research was carried out, are outlined in the IRB site: www.irbnet.org through which an approval for the study was obtained from.

No potential conflict of interest in the report is involved.

References

An, S. Kulm, G., and Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*. 7(2), 145-172.

Andrews, P. (2009). Comparative Studies of Mathematics Teachers' Observable Learning Objectives: Validating Low Inference Codes. *Educational Studies in Mathematics*, 71(2), 97-122.

Bair, S. L., & Rich, B. S. (2011). Characterizing the development of specialized mathematical content knowledge for teaching in algebraic reasoning and number theory. *Mathematical Thinking and Learning*, *13*(4), 292–321.

Baumert, J., et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47(1),* 133-180.

Bransford, J., Brown, A. & Cocking, R. (2000). *How People Learn*. Expanded Edition. Washington, DC: National Research Council.

Cai, J., Ding, M., and Wang, T. (2014). How do exemplary Chinese and U.S. mathematics teachers' view instructional coherence? *Educational Studies in Mathematics,* 85(2), 265-280.

Chapman, O. (2013). Investigating teachers' knowledge for teaching mathematics. *Journal of Mathematics Teacher Education, 16,* 237-243.

Charalambous, C.& Pitta-Pantazi, D. (2016). Perspectives on priority mathematics education unpacking and understanding a complex relationship linking teacher knowledge, teaching, and learning. In *Handbook of International Research in Mathematics Education*, 3rd Edition. Edited by Lyn D. English and David Kirshner. New York, NY: Routledge, 19-59.

Cronbach, L. (1951). Coefficient alpha and the internal structure of the tests. *Psychometrika*, 16(3), 297–334.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*, 61, 293–319.

Ferrini-Mundy, J., Floden, R. E., McCrory, R., Burrill, G., & Sandow, D. (2005). *Knowledge for teaching school algebra: Challenges in developing an analytic framework*. East Lansing, MI: Michigan State University, Knowledge of Algebra for Teaching Project.

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning*, *8*(1), 37–63.

Hemmi, K., & Ryve, A. (2015), Effective mathematics teaching in Finnish and Swedish teacher education discourses, *Journal of Mathematics Teacher Education*, 18(6), 501-521.

Hill, H., Schilling, S., & Ball, D. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105, 11–30.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education.* 39(4), 372-400.

Izsac, A., Jacobson, E., & de Araujo, Z. (2012). Measuring mathematical knowledge for teaching fractions with drawn quantities. *Journal for Research in Mathematics Education, 43(4),* 391-427.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Hillsdale, NJ: Erlbaum.

Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). The *TIMSS 2011 International Results in Mathematics.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Mccrory, R., Floden, R, Ferrini-Mundy, J., Reckase, M. & Senk, S. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education, 43(5),* 584-615.

Monk, D., & Rice, J. (1994). Multilevel teacher resource effects on pupil performance in secondary mathematics and science: The case of teacher subject-matter preparation. In R.G. Ehrenberg (Ed.), *Choices and consequences: Contemporary policy issues in education*. Ithaca, NY: ILR Press, 29-58.

Murphy, C. (2012). The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *Journal of Mathematics Teacher Education*, *15*(3), 187–206.

Nason, R., Chalmers, C., & Yeh, A. (2012). Facilitating growth in prospective teachers' knowledge: Teaching geometry in primary schools. *Journal of Mathematics Teacher Education*, *15*(3), 227–249.

Robitaille, D.F. & Travers, K.J. (1992). International studies of achievement in mathematics. In D.A. Grouws (Ed.), *Handbook of mathematics teaching and learning* (pp. 687–709). New York: Macmillan Publishing Company.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*, 255–281.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Sorto, M., Marshall, J., Luschel, T., and Carnoy, M. (2009). Teacher knowledge and teaching in Panama and Costa Rica: A comparative study in primary and secondary education. *Revista Latinoamerica de Investigacion en Matematica Educativa,* 12(2), 251-290.

Stein, M., Smith, M., Henningsen, M., & Silver, E. (2000). *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development*. Foreword by Deborah Ball. New York, NY: Teachers College Press.

Steinberg, T., Haymore, J., & Marks, R. (1985). Teacher's knowledge and structuring content in mathematics. Paper presented at the *Annual meeting of American Educational Research Association*, Chicago.

Stigler, J.W. & Perry, M. (1988). Cross-cultural studies of mathematics teaching and learning: Recent finding and new directions. In D. Grouws & T. Cooney (Eds.), *Effective mathematics teaching directions* (pp. 194–223). Reston, VA: National Council of Teachers of Mathematics.

Tatto, M. & Senk, S. (2011). The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics (TEDS-M), *Journal of Teacher Education*, 62 (2), 121-137

Tatto, M.T. (Ed.). (2013). The Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries. Technical report. Amsterdam: IEA.

Tchoshanov, M. A. (2011). Relationship between teacher content knowledge, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics, 76(2),* 141-164.

Tchoshanov, M., Cruz, M., Huereca, K., Shakirova, K., Shakirova, L., Ibragimova, E. (2016) Examination of lower secondary mathematics teachers' content knowledge and its connection to students' performance. *International Journal of Science and Mathematics Education,* DOI: 10.1007/s/10763-015-9703-9.

Wagner, S., Lee, H., & Ozgun-Koca, S. (1999). A comparative Study of the United States, Turkey, and Korea: Attitudes and Beliefs of Preservice Mathematics Teachers towards Mathematics, teaching mathematics, and their teacher preparation programs. *Proceedings of the Annual Meeting of Association of Mathematics Teacher Educators.* Chicago, IL.

Wang, J., & Lin, E. (2005). Comparative Studies on U.S. and Chinese Mathematics Learning and the Implications for Standards-Based Mathematics Teaching Reform. *Educational Researcher*, 34(5), 3-13.

