Proceedings of the 2020 IEEE/SICE
International Symposium on System Integration
Honolulu, Hawaii, USA, January 12-15, 2020

Function Approximation Technique Based Control for a Class of Nonholonomic Systems

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***Abstract*— A generic control method is proposed for a class of nonholonomic systems. A nonholonomic system to be controlled is first restructured in form of the combination of a linear system and the variation from the original system. This variation term is treated as a time-varying uncertainty and the stabilization problem for a nonholonomic system is reformulated as an adaptive control problem for linear system with time-varying uncertainty. This adaptive control problem is addressed by applying the function approximation technique. Specifically, the variation is parameterized with a chosen basis function weighted by unknown constant parameters. An update law is defined such that the parameters of the weighted basis function can be automatically determined and the variation between the auxiliary linear system and the original nonholonomic system can then be eliminated. The stability is established for the closed loop system formulated by the nonholonomic system and the constructed controller. The feasibility of the proposed control method is verified under simulations for two typical nonholonomic systems: the unicycle system and the rolling ball system.**

I. Introduction

Nonholonomic systems refer to those with non-integrable constraints. The need for the control of these systems arises in many applications such as wheeled robots, spherical robots, and underwater robots. The control problem commonly requires to steer the robotics systems from one to another configurations, including both their position and orientation. The position and orientation coupled together, form a nonholonomic constraint which makes the control problems difficult. It is because there exists no continuous state feedback for these systems that leads to asymptotic stability, according to the well known Brockett's theory [1].

Instead of using continuous state feedback to tackle the control problem for nonholonomic systems, in the literature, numerous approaches have been proposed, which can be roughly classified into the feedforward control (motion planning) and the feedback control. The feedforward control methods [2]–[10] commonly depend on sinusoidal, piecewise constant, or polynomial inputs and optimal control formulation. These open-looped methods work only when the system model is completely known and without perturbation.

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The feedback methods include time-varying algorithms [11]–[15] and the time-invariant algorithms based on discontinuous or switching control solutions [16]–[21]. A review of the above feedback controllers can be found in [22]. Recent works for the control of nonholonomic systems mainly focus on the model predictive control (MPC) methods [23]–[29]. Although the system outputs may finally converge to the desired under these methods, the control problems for nonholonomic systems are not completely solved due to following considerations. The smooth time-varying algorithms usually result to slow convergence. The switching based control methods yield fast convergence but they are usually based on the coordinate transformation which are not always straight forward. Although the MPC methods contribute significantly to explicitly optimize the overall performance of control systems, they are commonly time consuming and not applicable for real-time applications of robotic systems.

In this paper, we propose a novel control algorithm that is model independent and thus, applies to a wide class of nonholonomic systems. It has a rather low computational cost compared with the MPC based approaches. We construct the algorithm based on the function approximation technique, inspired by its application on the adaptive control [30]–[36]. Note that all nonholonomic systems can be expressed by the combination of an approximated square system, referring to the auxiliary system, and the variation from it. This variation term is treated as a time-varying uncertainty to the restructured square system.

Then the construction of the function approximation based controller has two parts. The first part is to control the auxiliary system, which is trivial. The second part is to eliminate the influence of the time varying uncertainty to the control process. We parameterize the uncertainty term with a set of chosen basis functions weighted by unknown parameters. Then we define the update laws such that the parameters of the weighted basis functions can be automatically determined and the variation between the auxiliary square system and the original nonholonomic system can be eliminated. By combining these two parts, we obtain a feedback controller that makes the nonholonomic system stable.

The rest of this paper is organized as follows. First, in Section II we state the control problem for the nonholonomic systems and illustrate the process for constructing a function approximation technique (FAT) controller. Also, in this section, we establish the stability for the closed loop system combined by the nonholonomic systems and the constructed FAT based controller. The feasibility of the proposed control method is verified under simulations in Section III. Finally,

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conclusions are drawn in Section IV.

II. Function approximation technique based feedback control

Given a nonholonomic system written in the state-space form

$$\dot{\boldsymbol{x}} = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{\xi}, \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^m$ represents for the state, $\boldsymbol{u} \in \mathbb{R}^n$ with $m > n$, represents for the input, and $\boldsymbol{\xi} \in \mathbb{R}^m$ for the external disturbance which is assumed to be bounded, construct an input $\boldsymbol{u}$ such that $\lim_{t\to\infty} \boldsymbol{x} = \boldsymbol{0}$.

As the number of inputs and that of the states are not matched, to square the control system, one introduces the auxiliary input $\boldsymbol{u}^* \in \mathbb{R}^m$ such that

$$\boldsymbol{u} = \boldsymbol{G}^*\boldsymbol{u}^*, \tag{2}$$

and the auxiliary matrix $\boldsymbol{G}^*$ is chosen to be a full rank $m \times n$ matrix, the weighted pseudoinverse of matrix $\boldsymbol{G}$

$$\boldsymbol{G}^* = (\boldsymbol{G}^\top \boldsymbol{W}\boldsymbol{G})^{-1}\boldsymbol{G}^\top \boldsymbol{W}, \tag{3}$$

where $\boldsymbol{W}$ is a constant matrix. An essential condition for the selection of matrix $\boldsymbol{G}^*$ is that the restructured system (4) must be controllable. Then the control system (1) is rewritten as

$$\dot{\boldsymbol{x}} = \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{u}^* + \boldsymbol{\xi}, \tag{4}$$

which is a square system (the number of the inputs equals that of the states).

One can restructure (4) as

$$\dot{\boldsymbol{x}} = \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{u}^* + \boldsymbol{\xi} + \boldsymbol{u}^* - \boldsymbol{u}^* = \boldsymbol{u}^* + \boldsymbol{d}, \tag{5}$$

where

$$\boldsymbol{d}(\boldsymbol{x}, t, \boldsymbol{u}^*) = (\boldsymbol{G}\boldsymbol{G}^* - \boldsymbol{I})\boldsymbol{u}^* + \boldsymbol{\xi}. \tag{6}$$

Note that through the above rearrangement, the original system (1) is restructured as the combination of two parts, a trivial linear system $\dot{\boldsymbol{x}} = \boldsymbol{u}^*$ referring to the auxiliary system, and $\boldsymbol{d}$, which can be viewed as an uncertainty term to the auxiliary system. Thus the control problem for a nonholonomic system is reformulated to the adaptive control problem for a linear system with time-varying uncertainties, which is stated as constructing $\boldsymbol{u}^*$ such that $\lim_{t\to\infty} \boldsymbol{x} = \boldsymbol{0}$, with $\boldsymbol{d}$ unknown.

Compared with adaptive control problem for systems with parametric errors, it is challenging for systems with time-varying uncertainty where traditional techniques such as the model reference adaptive control (MRAC) are not feasible. To tackle this problem, we adopt the FAT based adaptive control design [30]–[33], [37]–[41]. Compared with the traditional MRAC, the advantage of the FAT based control is in the representation of the time-varying uncertainties by a set of given basis functions weighted by a set of unknown constant parameters. Thus the problem of eliminating the influence of the uncertainty terms is transformed to the estimation of parametric errors. Then Lyapunov designs are applied to derive proper update laws adjusting the estimates of the unknown parameters.

To control (5), the effect of the uncertainty term $\boldsymbol{d}$ to the control system needs to be eliminated. Note that the uncertainty term varies with respect to time t and the state $\boldsymbol{x}$. We approximate $\boldsymbol{d}$ as

$$\boldsymbol{d}(\boldsymbol{x}, t, \boldsymbol{u}^*) = \sum_{i=0}^{N} \boldsymbol{d}_i \psi_i(\boldsymbol{x}, t) + \boldsymbol{\epsilon}, \tag{7}$$

where $\boldsymbol{d}_i$ is constant and ψ_i consists of $\boldsymbol{x}$ and t, and $\boldsymbol{\epsilon}$, referring to the approximation error, describes the deviation between the uncertainty $\boldsymbol{d}$ and the weighted basis functions. Note that when $N \to \infty$, the approximation error $\boldsymbol{\epsilon}$ would vanish.

To control (5), the effect of the uncertainty term $\boldsymbol{d}$ to the system needs to be eliminated. Note that the uncertainty term varies with respect to time t and the state $\boldsymbol{x}$ (the input $\boldsymbol{u}$ can also be expressed by t and $\boldsymbol{x}$). However, at any moment (i.e., $t = 1$, $t = 2$, ...) $\boldsymbol{d}$ is a constant. We use weighted basis functions to approximate $\boldsymbol{d}$ at each moment such that

$$\dot{\boldsymbol{x}} = \boldsymbol{u}^* + \sum_{i=0}^{N} \boldsymbol{d}_i \psi_i(\boldsymbol{x}, t) + \boldsymbol{\epsilon}, \tag{8}$$

where $\boldsymbol{d}_i$, referring to the plant parameter, is constant, ψ_i, referring to the basis function, consists of $\boldsymbol{x}$ and t, and $\boldsymbol{\epsilon}$, referring to the approximation error, describes the deviation between the uncertainty term $\boldsymbol{d}$ and its approximation, the weighted basis functions.

The control of (8) requires the unknown plant parameters $\boldsymbol{d}_i$ to be identified. For this purpose, these plant parameters $\boldsymbol{d}_i$ at each time t are estimated by $\hat{\boldsymbol{d}}_i(t)$, referring to the control parameter. The change of $\hat{\boldsymbol{d}}_i(t)$, with respect to time, is governed by an update law that we are to define in the construction of a feedback controller $\boldsymbol{u}^*$ from the following process.

To construct $\boldsymbol{u}^*$ that steers $\boldsymbol{x}$ to zero and an update law that defines $\hat{\boldsymbol{d}}_i(t)$, a feasible Lyapunov candidate function would be

$$V = \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{x} + \frac{1}{2}\sum_{i=0}^{N}\Big(\hat{\boldsymbol{d}}_i(t) - \boldsymbol{d}_i\Big)^\top\Big(\hat{\boldsymbol{d}}_i(t) - \boldsymbol{d}_i\Big), \tag{9}$$

which combines both the state $\boldsymbol{x}$ and the estimation error $\hat{\boldsymbol{d}}_i(t) - \boldsymbol{d}_i$ between the plant parameters $\boldsymbol{d}_i$ and their estimates $\hat{\boldsymbol{d}}_i(t)$. Note that in (9), the parameters $\boldsymbol{d}_i$ are constants, which implies $\dot{\boldsymbol{d}}_i = \boldsymbol{0}$. Therefore the derivative of the Lyapunov function candidate is calculated as

$$\dot{V} = \boldsymbol{x}^\top \dot{\boldsymbol{x}} + \sum_{i=0}^{N}\Big(\hat{\boldsymbol{d}}_i(t) - \boldsymbol{d}_i\Big)^\top \dot{\hat{\boldsymbol{d}}}_i(t). \tag{10}$$

To cancel the terms with $\boldsymbol{d}_i$, define the update law

$$\dot{\hat{\boldsymbol{d}}}_i(t) = \boldsymbol{x}\psi_i(\boldsymbol{x}, t), \tag{11}$$

which leads to

$$\dot{V} = -\boldsymbol{x}^\top\Big(\boldsymbol{u}^* + \sum_{i=0}^{N}\hat{\boldsymbol{d}}_i(t)\psi_i(\boldsymbol{x}, t) + \boldsymbol{\epsilon}\Big).$$

The approximation error $\boldsymbol{\epsilon}$ needs to be considered in the construction of the auxiliary input $\boldsymbol{u}^*$. Select $\boldsymbol{u}^*$ as

$$\boldsymbol{u}^* = \boldsymbol{u}_x^* + \boldsymbol{u}_\epsilon^*, \tag{12}$$

where $\boldsymbol{u}_\epsilon^*$ is to cover the effect of $\boldsymbol{\epsilon}$. Constructing

$$\boldsymbol{u}_x^* = -\boldsymbol{K}\boldsymbol{x} - \sum_{i=0}^{N} \hat{\boldsymbol{d}}_i(t)\psi_i(\boldsymbol{x}, t), \tag{13}$$

where $\boldsymbol{K}$ is a positive matrix, yields

$$\dot{V} = -\boldsymbol{x}^\top \boldsymbol{K}\boldsymbol{x} - \boldsymbol{x}^\top \Big(\boldsymbol{u}_\epsilon^* + \boldsymbol{\epsilon}\Big). \tag{14}$$

Then one designs a robust control law for $\boldsymbol{u}_\epsilon^*$ to cover the effect of $\boldsymbol{\epsilon}$. Denote the components of $\boldsymbol{\epsilon} \in \mathbf{R}^m$ to be ϵ_j, where $j = 1, 2, ..., m$. Suppose ϵ_j is bounded and its variation bound is available, i.e., there exists $\delta_j > 0$ such that $\|\epsilon_j\| \leq \delta_j$. Then selecting $u_{\epsilon j}^*$ as

$$u_{\epsilon j}^* = -\delta_j \operatorname{sgn}(x_j) \tag{15}$$

yields

$$\dot{V} = -\boldsymbol{x}^\top \boldsymbol{K}\boldsymbol{x} + \sum_{j=0}^{m} \Big(x_j \epsilon_j - \delta_j \|x_j\|\Big). \tag{16}$$

Note that

$$\sum_{j=0}^{m} \Big(x_j \epsilon_j - \delta_j \|x_j\|\Big) \leq 0 \tag{17}$$

holds true when ϵ_j is bounded by δ_j. Therefore, $\dot{V} \leq 0$ is guaranteed under the control law

$$\boldsymbol{u} = \boldsymbol{G}^*(\boldsymbol{u}_x^* + \boldsymbol{u}_\epsilon^*). \tag{18}$$

The convergence of the state $\boldsymbol{x}$ is achieved while the effect of the approximation error $\boldsymbol{\epsilon}$ is covered. Thus, (18) together with the update law (11) formulates the FAT based controller for the nonholonomic systems.

III. Case study

In this section, we test the validity of the constructed controller applied on two typical nonholonomic system, the unicycle system and the rolling ball system. We choose a set of polynomial basis functions for estimating the variation $\boldsymbol{d}$ expressed by (6), that is,

$$\psi_i(\boldsymbol{x}, t) = t^i, \tag{19}$$

where the accuracy of the approximation can be adjusted by changing the order of the polynomial function.

A. Unicycle

A unicycle is a vehicle with a single orientable wheel. Its configuration is described by

$$\boldsymbol{x} = (x, y, \theta), \tag{20}$$

where x and y are the Cartesian coordinates of the contact point of the wheel with the ground, and the angle θ is the orientation of the wheel with respect to the x-axis. The input to the system is defined by

$$\boldsymbol{u} = (v, \omega), \tag{21}$$

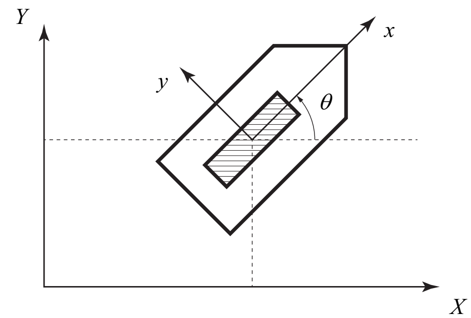


Fig. 1. Unicycle system

where v and ω are respectively the linear and angular velocities of the vehicle. The kinematic model of the unicycle is then derived as

$$\dot{\boldsymbol{x}} = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{22}$$

The control problem is then described as constructing an input $\boldsymbol{u}$ such that $\lim_{t\to\infty} \boldsymbol{x} = \boldsymbol{0}$. For the construction of input $\boldsymbol{u}$, let us introduce the auxiliary input $\boldsymbol{u}^*$, where

$$\boldsymbol{u} = \boldsymbol{G}^*\boldsymbol{u}^*, \tag{23}$$

By selecting $\boldsymbol{G}^*$ as (3) where $\boldsymbol{W}$ is an identity matrix, the state equation is rewritten as

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{3} \boldsymbol{g}_i^* u_i^*, \tag{24}$$

where

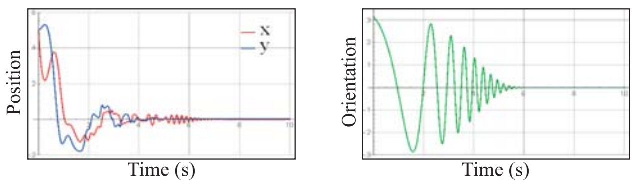
$$\boldsymbol{g}_1^* = \begin{bmatrix} \cos^2\theta \\ \frac{1}{2}\sin 2\theta \\ 0 \end{bmatrix}, \; \boldsymbol{g}_2^* = \begin{bmatrix} \frac{1}{2}\sin 2\theta \\ \sin^2\theta \\ 0 \end{bmatrix}, \; \boldsymbol{g}_3^* = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

For the application of the proposed FAT based controller, the reformulated system is required to be controllable. To test the validity of the chosen $\boldsymbol{G}^*$ matrix, let us then check the local controllability around the equilibrium point $\boldsymbol{x}_o = (0, 0, 0)$ for the reformulated unicycle system (24). The Lie brackets calculated for (24) around the equilibrium point $\boldsymbol{x} = \boldsymbol{x}_o$ are

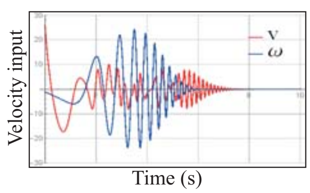
$$\begin{aligned} &\mathcal{L}_1 = \boldsymbol{g}_1^*, \mathcal{L}_2 = \boldsymbol{g}_2^*, \mathcal{L}_3 = \boldsymbol{g}_3^*, \\ &\mathcal{L}_4 = [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*], \mathcal{L}_5 = [\boldsymbol{g}_1^*, \boldsymbol{g}_3^*], \mathcal{L}_6 = [\boldsymbol{g}_2^*, \boldsymbol{g}_3^*], \\ &\mathcal{L}_7 = [\boldsymbol{g}_1^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]], \mathcal{L}_8 = [\boldsymbol{g}_2^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]], \\ &\mathcal{L}_9 = [\boldsymbol{g}_3^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]], \\ &\mathcal{L}_{10} = [\boldsymbol{g}_1^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_3^*]], \mathcal{L}_{11} = [\boldsymbol{g}_2^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_3^*]], \\ &\mathcal{L}_{12} = [\boldsymbol{g}_3^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_3^*]], \\ &\mathcal{L}_{13} = [\boldsymbol{g}_1^*, [\boldsymbol{g}_2^*, \boldsymbol{g}_3^*]], \mathcal{L}_{14} = [\boldsymbol{g}_2^*, [\boldsymbol{g}_2^*, \boldsymbol{g}_3^*]], \\ &\mathcal{L}_{15} = [\boldsymbol{g}_3^*, [\boldsymbol{g}_2^*, \boldsymbol{g}_3^*]]. \end{aligned}$$

As $\operatorname{rank}([\mathcal{L}_1|\mathcal{L}_2|...]) = 3$, the reconstructed unicycle system is locally accessible at the equilibrium point. Because the system is driftless, local accessibility implies local controllability.

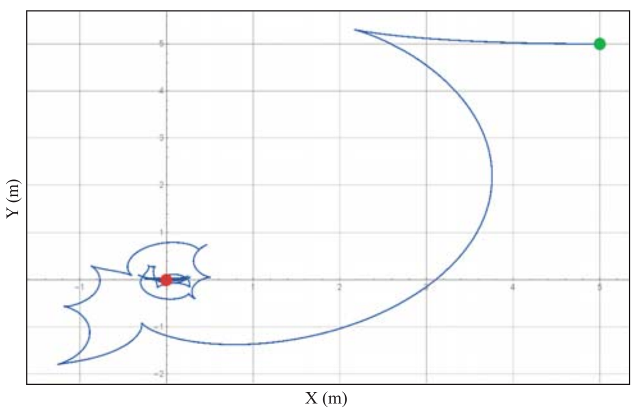
Knowing that the reconstructed unicycle system (24) is locally controllable, we apply the constructed control law, where the gain matrix is selected as $\boldsymbol{K} = \text{diag}\{5, 5, \frac{e^{(t+0.5)}}{50}\}$. Note that the selection of exponential components in $\boldsymbol{K}$ is to increase the converging speed of the output signals. The system responses x, y, and θ converge to desired values, as illustrated in Fig. 2(a). The initial conditions of x, y, and θ are specified as 5 m, 5 m, and π rad. The input signals are shown by Fig. 2(b). The trace for the unicycle is plotted in Fig. 2(c). The performance of the controller can be improved by the selection of basis functions other than polynomials.



(a) Trajectories for the position (left) and the orientation (right) of a rolling ball under the FAT control.



(b) The input signals under the FAT control.



(c) Trace of a rolling ball on a flat plane under the FAT control.

Fig. 2. Simulation results for the unicycle system on a flat plane under the FAT control.

B. Rolling ball

The changes for the position and orientation of a rolling ball are coupled together and thus, form a nonholonomic constraint. Define the configuration of a rolling ball by

$$\boldsymbol{x} = (u_b, v_b, u_o, v_o, \psi), \tag{25}$$

where u_b, v_b represent for the displacement of the geometric center of the ball along X-axis and Y-axis, u_o, v_o, and ψ are a set of special Eular angles that describe the orientation of the ball. The input to this system is defined by

$$\boldsymbol{u} = (\omega_x, \omega_y, \omega_z), \tag{26}$$

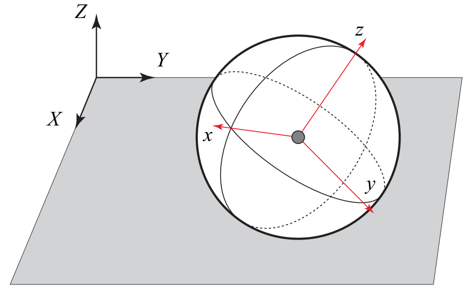


Fig. 3. A rolling ball on a flat plane

where ω_x, ω_y, and ω_z are respectively the angular velocities of the ball. The contact kinematics of the rolling ball on a flat plane is described by Montana's equations [4], which in our parameterization take the following form [42]

$$\dot{\boldsymbol{x}} = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}, \tag{27}$$

where

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} 0 & R & 0 \\ -R & 0 & 0 \\ -\sin\psi/\cos v_o & -\cos\psi/\cos v_o & 0 \\ -\cos\psi & \sin\psi & 0 \\ -\sin\psi\tan v_o & -\cos\psi\tan v_o & -1 \end{bmatrix}. \tag{28}$$

Note that the geometric singularities exist in the above system when $v_o = \frac{\pi}{2}$.

The control problem is then stated as constructing an input $\boldsymbol{u}$ such that $\lim_{t\to\infty} \boldsymbol{x} = \boldsymbol{0}$. For the construction of input $\boldsymbol{u}$, let us introduce the auxiliary input $\boldsymbol{u}^*$, where

$$\boldsymbol{u} = \boldsymbol{G}^*\boldsymbol{u}^*, \tag{29}$$

By selecting $\boldsymbol{G}^*$ as (3) where $\boldsymbol{W}$ is an identity matrix, one rewrites the state equation as

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{5} \boldsymbol{g}_i^* u_i^*, \tag{30}$$

where

$$\boldsymbol{g}_1^* = \begin{bmatrix} \frac{2R^2\left((R^2+\cos^2\psi)\cos^2 v_o+\sin^2\psi\right)}{\gamma_1\gamma_2} \\ \frac{R^2\sin 2\psi\sin^2 v_o}{\gamma_1\gamma_2} \\ \frac{2R\cos\psi\cos v_o}{\gamma_2} \\ \frac{R\sin\psi}{\gamma_1} \\ 0 \end{bmatrix},$$

$$\boldsymbol{g}_2^* = \begin{bmatrix} \frac{R^2\sin 2\psi\sin^2 v_o}{\gamma_1(2+R^2(\cos 2v_o+1))} \\ \frac{2R^2\left((R^2+\cos^2\psi)\cos^2 v_o+\sin^2\psi\right)}{\gamma_1\gamma_2} \\ \frac{2R\sin\psi\cos v_o}{\gamma_2} \\ \frac{R\cos\psi}{\gamma_1} \\ 0 \end{bmatrix},$$

$$\boldsymbol{g}_3^* = \begin{bmatrix} \frac{2R\cos\psi\cos v_o}{\gamma_2} \\ \frac{2R\cos v_o\sin\psi}{\gamma_2} \\ \frac{2}{\gamma_2} \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{g}_4^* = \begin{bmatrix} \frac{R\sin\psi}{\gamma_1} \\ \frac{R\cos\psi}{\gamma_1} \\ 0 \\ \frac{1}{\gamma_1} \\ 0 \end{bmatrix}, \boldsymbol{g}_5^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

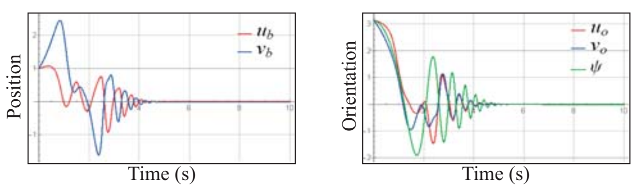
and $\gamma_1 = 1+R^2, \gamma_2 = 2+R^2(\cos 2v_o+1)$. Note that $\gamma_1 > 0$ and $\gamma_2 > 0$ such that singularities ($v_o = \frac{\pi}{2}$) in the original system can be removed.

For the application of the proposed FAT based controller, the reformulated system is required to be controllable. To test the validity of the chosen $\boldsymbol{G}^*$ matrix, the local controllability around the equilibrium point $\boldsymbol{x} = (0,0,0,0,0)$ for the rolling ball system is checked below. At the equilibrium point $\boldsymbol{x} = \boldsymbol{x}_o$. The Lie brackets calculated for the control system are

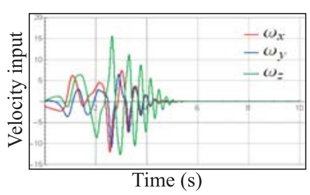
$$\begin{aligned}
&\mathcal{L}_1 = \boldsymbol{g}_1^*, \mathcal{L}_2 = \boldsymbol{g}_2^*, \mathcal{L}_3 = \boldsymbol{g}_3^*, \mathcal{L}_4 = \boldsymbol{g}_4^*, \mathcal{L}_5 = \boldsymbol{g}_5^*,\\
&\mathcal{L}_6 = [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*], \mathcal{L}_7 = [\boldsymbol{g}_1^*, \boldsymbol{g}_3^*], \mathcal{L}_8 = [\boldsymbol{g}_1^*, \boldsymbol{g}_4^*], \ldots\\
&\mathcal{L}_{16} = [\boldsymbol{g}_1^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]], \mathcal{L}_{17} = [\boldsymbol{g}_2^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]], \ldots\\
&\mathcal{L}_{71} = [\boldsymbol{g}_1^*, [\boldsymbol{g}_2^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]]], \mathcal{L}_{72} = [\boldsymbol{g}_2^*, [\boldsymbol{g}_2^*, [\boldsymbol{g}_1^*, \boldsymbol{g}_2^*]]], \ldots
\end{aligned}$$

As $\text{rank}([\mathcal{L}_1|\mathcal{L}_2|\ldots]) = 5$ at the equilibrium point, the system under consideration is locally accessible at the equilibrium point. For driftless system, local accessability implies local controllability.

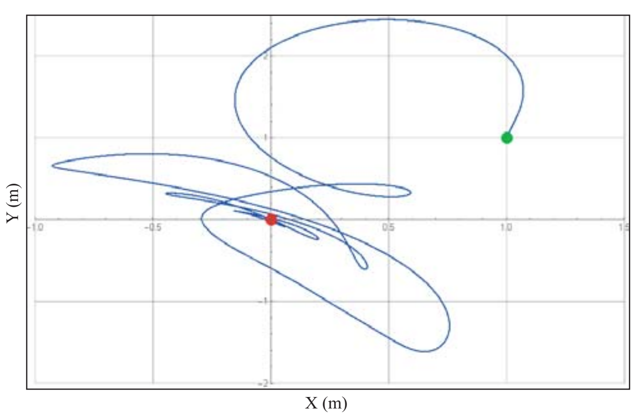
As the reconstructed system (30) is locally controllable, simulations are conducted as follows under the proposed FAT based controller. The gain matrix $\boldsymbol{K}$ is selected as $\boldsymbol{K} = \text{diag}\{\frac{e^{(t+0.5)}}{10}, \frac{e^{(t+0.5)}}{10}, \frac{e^{(t+0.5)}}{10}, \frac{e^{(t+0.5)}}{2}, \frac{e^{(t+0.5)}}{25}\}$. The trajectories for the full configuration of the rolling ball, including its position $u_b(t)$, $v_b(t)$, and its orientation $u_o(t)$, $v_o(t)$, $\psi(t)$, converge to zero, as illustrated in Fig. 4(a). The initial conditions of the system state $u_b(t)$, $v_b(t)$, $u_o(t)$, $v_o(t)$, and $\psi(t)$ are specified as 1 m, 1 m, π rad, π rad, and π rad. The input signals are also shown by Fig. 4(b). The trace for the geometric center of the rolling ball is plotted in Fig. 4(c). The performance of the controller can be improved by the selection of basis functions other than polynomials.



(a) Trajectories for the position (left) and the orientation (right) of a rolling ball under the FAT control.



(b) The input signals under the FAT control.



(c) Trace of a rolling ball on a flat plane under the FAT control.

Fig. 4. Simulation results for a rolling ball on a flat plane under the FAT control.

IV. Conclusions

An FAT based controller has been proposed for a class of nonholonomic systems. By introducing an auxiliary input the dimension of which equals that of the system state, a nonholonomic system was restructured in the form of the combination of a controllable linear system (auxiliary system) and the variation from the original system, which is treated as system uncertainties.

In the control process, the uncertainty term was replaced by its approximation as a chosen basis function weighted by constant parameters to be determined. These unknown plant parameters are estimated at each instant, denoted by the adjustable control parameters using a defined update law. Thus the influence to the control process caused by the uncertainty can be eliminated.

The stability was established for the closed loop system formulated by the nonholonomic system and the constructed controller. The feasibility of the proposed control method was verified under simulations for the point stabilization problem of a linear system, a nonlinear underactuated system, and a nonholonomic system, respectively.

Acknowledgements

This research was supported, in part, by the Japan Science and Technology Agency, the JST Strategic International Collaborative Research Program, Project No. 18065977.

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