International Conference on Noncommutative Geometry, Analysis on Groups, and Mathematical Physics

Ghent Analysis and PDE Center, Ghent University (Online Event)

Conference Webpage

Organizing Committee

Gihyun Lee (Ghent University, Belgium)
Michael Ruzhansky (Ghent University, Belgium)
Kanat Tulenov (Ghent University, Belgium)

Join Zoom Meeting

Meeting ID: 895 8541 1857

Passcode: 496026

February 26–27, 2024

Contents

Timetable

The schedule is based on Central European Time (UTC+1).

	26 Feb (Monday)
09:30-10:00	Bourne
10:05-10:35	Yu
10:40-10:50	Short Break
10:50-11:20	Vaes
11:25-11:45	Coffee Break
11:45-12:15	Bekjan
12:20-14:00	Lunch
14:00-14:30	Fulsche
14:35-15:05	Park
15:10-15:40	Bikchentaev
15:45-16:15	Coffee Break
16:15-16:45	Kai
16:50-17:20	Mei
17:25-17:55	Lafleche

	27 Feb (Tuesday)
09:00-09:30	Nessipbayev
09:35-10:05	Xiong
10:10-10:40	Austad
10:45-11:15	Coffee Break
11:15-11:45	Youn
11:50-12:20	Voigt
12:25-14:00	Lunch
14:00-14:30	Christensen
14:35-15:05	Kim
15:10-15:40	Francis
15:45-16:15	Coffee Break
16:15-16:45	Klisse
16:50-17:20	McDonald
17:25-17:55	Majid

Conference Webpage

Join Zoom Meeting

Meeting ID: 895 8541 1857
Passcode: 496026

Titles and Abstracts

Quantum metrics on crossed products with groups of polynomial growth

Tuesday
27 Feb
10:10–10:40

Are Austad
University of Southern Denmark

In Rieffel's theory of compact quantum metric spaces, an important source of examples comes from discrete groups equipped with length functions. We show how to combine the compact quantum metric space structure of a finitely generated group of polynomial growth with a compact quantum metric space it acts on to construct a quantum metric space structure on the crossed product. Moreover, the construction is compatible with the unbounded Kasparov product. This is joint work with Jens Kaad and David Kyed.

Atomic decomposition of noncommutative martingales

Monday
26 Feb
11:45–12:15

Turdebek N. Bekjan
Astana IT University

Atomic decomposition plays a fundamental role in the classical martingale theory and harmonic analysis. For instance, atomic decomposition is a powerful tool for dealing with duality theorems, interpolation theorems and some fundamental inequalities both in martingale theory and harmonic analysis. Atoms for martingales are usually defined in terms of stopping times. Unfortunately, the concept of stopping times is, up to now, not well-defined in the generic noncommutative setting (there are some works on this topic, for example,see the works of Attal and Coquio). We note, however, that atoms can be defined without help of stopping times.

In this talk, we show that the atomic decomposition for the Hardy spaces h_1 and $\mathcal{H}_1$ is valid for noncommutative martingales. We also present recent results on atomic decomposition.

Zero-trace commutators of measurable operators

Monday
26 Feb
15:10–15:40

Airat Bikchentaev
Kazan Federal University

Let τ be a faithful normal semifinite trace on a von Neumann algebra $\mathcal{M}$, I be the unit of $\mathcal{M}$, $S(\mathcal{M},\tau)$ be the *-algebra of all τ-measurable operators, $L_1(\mathcal{M},\tau)$ be the Banach space of all τ-integrable operators. We present a new proof of the following generalization of Putnam Theorem (1951): a positive selfcommutator $[A^*,A]$ ($A \in S(\mathcal{M},\tau)$) cannot have the inverse in $\mathcal{M}$. If the trace τ is infinite, then the positive selfcommutator $[A^*,A]$ ($A \in S(\mathcal{M},\tau)$) cannot have the form $\lambda I + K$, where λ is a non-zero complex number and an operator K is τ-compact. Let $A, B \in S(\mathcal{M},\tau)$ and $[A,B] \in L_1(\mathcal{M},\tau)$. *Question*: under which conditions $\tau([A,B]) = 0$? If $X \in S(\mathcal{M},\tau)$, $Y = Y^3 \in \mathcal{M}$ and $[X,Y] \in L_1(\mathcal{M},\tau)$, then $\tau([X,Y]) = 0$. If $A^2 = A \in S(\mathcal{M},\tau)$ and $[A^*,A] \in L_1(\mathcal{M},\tau)$, then $\tau([A^*,A]) = 0$. If a partial isometry U lies in $\mathcal{M}$ and $U^n = 0$ for some integer $n \geq 2$, then the operator U^{n-1} is a commutator and if $U^{n-1} \in L_1(\mathcal{M},\tau)$, then $\tau(U^{n-1}) = 0$.

Operator algebras and index theory in quantum walks

Monday
26 Feb
09:30–10:00

Chris Bourne
Nagoya University

Quantum walks are quantum analogues of random walks and can be used to implement certain quantum computational algorithms. I will give an introduction to quantum walk systems and how we can use techniques from crossed product C*-algebras to compute the essential spectrum of quantum walk unitaries. Time permitting, I will also discuss how we can use index theory to characterise topologically stable bound states of quantum walks with additional symmetries. This is partly based on arXiv:2211.10601 as well as work-in-progress with S. Richard and Y. Tanaka.

Tracial weights on topological graph C*-algebras

Tuesday
27 Feb
14:00–14:30

Johannes Christensen
KU Leuven

There is an abundance of C*-algebras which can be realised as the C*-algebras of topological graphs, like C*-algebras arising from crossed products of $\mathbb{Z}$ by homeomorphisms, C*-algebras arising from directed graphs, Kirchberg algebras satisfying the UCT and AF-algebras. Since C*-algebras of topological graphs are such a rich source of examples, it is natural to study traces on them. This was originally done by Schafhauser, who described gauge-invariant tracial states in terms of certain measures on the space of vertices of the graph, and conjectured that all tracial states are gauge-invariant for so-called *free* topological graphs.

For non-unital C*-algebras it is natural to study the generalisation of tracial states called *tracial weights*. In this talk, I will provide a graph-theoretical condition on topological graphs which is equivalent to the existence of tracial weights that are not gauge-invariant. Using this condition, I will prove that all tracial weights on a free topological graph are gauge-invariant. This result in particular provides an affirmative answer to Schafhauser's conjecture. I will also deduce a graph-theoretical condition which is equivalent to the existence of tracial states.

Normal forms in complex b^k-geometry

Tuesday
27 Feb
15:10–15:40

Michael Francis
University of Western Ontario

The b-tangent bundle (terminology due to Melrose) is defined so that its sections are smooth vector fields on a base manifold tangent along a given hypersurface. Complex b-manifolds, studied by Mendoza, are defined just like ordinary complex manifolds, replacing the usual tangent bundle by the b-tangent bundle. Recently, a Newlander-Nirenberg theorem for b-manifolds was obtained by Francis-Barron, building on Mendoza's work. This talk will discuss the extension of the latter result to the setting of b^k-geometry for $k > 1$. The original approach to b^k-geometry is due to Scott. A different approach that allows for global holonomy phenomena not present in Scott's framework was introduced by Francis and, independently, by Bischoff-del Pino-Witte.

What is quantum harmonic analysis?

Monday
26 Feb
14:00–14:30

Robert Fulsche
Leibniz University Hannover

There is nowadays a large collection of mathematics which may be described as *Quantum Harmonic Analysis* (QHA). In principle, this term could refer to any kind of mathematics connecting tools from harmonic analysis and their applications in quantum mechanics. I will talk about QHA in a rather strict sense, referring to the subject that was initiated by physicist Reinhard Werner in his 1984 paper "Quantum harmonic analysis on phase space".

The goal of Werner's QHA is, loosely speaking, establishing operations from classical harmonic analysis, which are well-known for functions (shifts, convolutions, Fourier transforms, ...), for operators, and to investigate how these notions interact with each other. In this approach, he used many mathematical objects which were previously known, such as Weyl systems, integrated representations and Wigner transforms, added some new objects (mainly the *convolution of two operators*) and put them together in a very convenient framework.

QHA, originating from works in theoretical physics, has been ignored by mathematicians for a long time. Only in the last few years, people working in time-frequency analysis as well as operator theory found a liking in this subject, which lead to some important applications of the theory.

In my talk, I will first give a very quick introduction to the tools, methods and slang of QHA. Once these basics are settled, I will try to explain some of the motivations to deal with QHA - since I am no physicist, I will restrict myself mostly to operator theory and omit other noteworthy fields. A few of the many important applications of QHA will be shown.

Schatten properties of commutators on noncommutative euclidean space

Monday
26 Feb
16:15–16:45

Zeng Kai
University of Franche-Comté

In this talk, we study Fourier multiplier commutators on noncommutative euclidean space $\mathbb{R}^d_\theta$. We will characterise their Schatten p-class membership by that of their symbols in the associated Besov space. In addition, we show a formula on the Dixmier trace, which also gives us a characterization of the weak Schatten p-class membership of these commutators by a Sobolev space.

NC convexity and operator system duality

Tuesday
27 Feb
14:35–15:05

Se Jin Kim
KU Leuven

Operator systems are linear spaces of bounded operators on a fixed Hilbert space that is closed under the involution operation. If the operators in this operator system commute, this is what is known in the literature as a function system. A foundational result of Kadison demonstrates that function systems are categorically dual to the category of compact convex sets. Here the function system can be thought of as the scalar valued affine functions over this convex set. We may therefore think of operator systems in general as a kind of non-commutative function system.

NC convex sets were introduced first in the matricial case by Webster–Winkler in 2008, and in a more general unital setting by Davidson–Kennedy in 2019, and finally in general by myself, Matt Kennedy, and Nicholas Manor. These nc convex sets are geometric objects closed under a certain matricial analogue of convex combinations. Surprisingly, Kadison's orginal duality theorem goes through completely in this far more general setting, allowing us use intuition that we have for convex sets in this new category. In this talk we will explore this duality in more detail, and explain how one may use commutative intuition to derive facts about operator systems using this nc convex algebraic geometry.

Harmonic decompositions of cocycles with coefficients in Banach spaces

Tuesday
27 Feb
16:15–16:45

Mario Klisse
KU Leuven

In recent years the study of cohomological aspects of linear representations of topological groups on Banach spaces has gained increased attention. The topic touches important notions like fixed point properties, Kazhdan's property (T), amenability, a-T-menability and the Haagerup property (and their generalizations), as well as lp-cohomology. The aim of this talk will be to first introduce analogues to classical notions from group theory (such as harmonicity of functions on groups) in the Banach space setting. After that we will demonstrate their usefulness in the context of continuous group cohomology by studying harmonic decompositions of cocycles. This will allow us to complement a number of existing results from the literature.

On some nocommutative analogues of Sobolev and optimal transport distances

Monday
26 Feb
17:25–17:55

Laurent Lafleche
ENS Lyon

In the context of proving the semiclassical mean-field limit from the N-body Schrödinger equation to the Hartree-Fock and Vlasov equations, a crucial component is obtaining inequalities uniform in the Planck constant and the number of particles. These inequalities can be viewed as a noncommutative analogue of those obtained in the corresponding models of classical statistical mechanics in the phase space.

In particular, one can define the analogue of Sobolev spaces on the phase space in terms of Schatten norms of commutators and the analogue of the classical optimal transport distances in terms of trace of operators. In this context, we will see that the quantum analogue of Sobolev inequalities yields uncertainty inequalities concerning the Wigner–Yanase skew information, and that the latter also plays a significant role in controlling a quantum optimal transport "self-distance".

Quantum geodesics on graphs

Tuesday
27 Feb
17:25–17:55

Shahn Majid
Queen Mary University of London

Quantum geodesics are a coordinate invariant tool to study the properties of a quantum Riemannian geometry. The latter is a framework for noncommutative geometry starting from a quantum metric in the tensor square of a bimodule of 1-forms over the algebra. Associated to such data and a quantum Levi-Civita (or other) connection, quantum geodesics are solutions of a certain flow equation which classically would an evolution equation for a wave function whose modulus square behaves like the a fluid density for particles moving on geodesics. The talk itself will be at an algebraic level describing the quantum geometry and the construction of the flow equations, and I will show some solutions on a finite-dimensional Hilbert space based on a small graph taken from recent work with Beggs. I will explain some of the issues for the infinite-dimensional ℓ^2 case which could be of interest for analysis. If time I will also mention the quantum Riemannian geometry behind certain spectral triples in the Connes sense, again at a pre-Hilbert space (i.e. pre-spectral triple) level.

Nonlinear PDE on noncommutative $\mathbb{R}^d$

Tuesday
27 Feb
16:50–17:20

Edward McDonald
Penn State University

Noncommutative Euclidean space is one of the model examples in noncommutative geometry and analysis. I will describe the basic features of function spaces and PDE on these objects, including some of the surprising features of nonlinear PDE in noncommutative space.

L^p-unconditional partitions of free group von Neumann algebras

Monday
26 Feb
16:50–17:20

Tao Mei
Baylor University

Let $\mathbb{F}_n, 2 \leq n \leq \infty$, be the non-abelian free group of n-free generators, and $\mathbb{F}_n^{(i)}$ be the subsets of $\mathbb{F}_n$ consisting of reduced words starting with the i-th generator. The partition $\mathbb{F}_n = \cup_{1\leq i\leq n}\mathbb{F}_n^{(i)} \cup \{e\}$ implies the well-known nonameanablility of $\mathbb{F}_n$. In a recent joint work with E. Ricard, we show that this partition is unconditional with respect to the noncommutative L^p-norm. This implies that the group von Neumann algebra of $\mathbb{F}_\infty$ admits a L^p-unconditional partition with infinitely many components that satisfy a geometrical paradoxical property.

It is a mystery whether the group von Neumann algebra of $\mathbb{F}_2$ (or $\mathbb{F}_n$ for any finite n) admits such a partition. In this talk, I wish to introduce recent progress in this direction. Part of the talk is based on joint work with Z. Liu, S. Yin and joint work with E. Ricard, Q. Xu.

The Gelfand–Phillips and Dunford–Pettis type properties in bimodules of measurable operators

Tuesday
27 Feb
09:00–09:30

Yerlan Nessipbayev
University of New South Wales

We characterize noncommutative symmetric spaces $E(\mathcal{M}, \tau)$ affiliated with a semifinite von Neumann algebra $\mathcal{M}$ equipped with a faithful normal semifinite trace τ on a Hilbert space having the Gelfand–Phillips property and the WCG–property. The list of their relations with other classical structural properties (such as the Dunford–Pettis property, the Schur property and their variations) is given in the setting of noncommutative symmetric spaces.

Random covariant quantum channels

Monday
26 Feb
14:35–15:05

Sang-Jun Park
Toulouse III - Paul Sabatier University

The group symmetries inherent in quantum channels often make them tractable and applicable to various problems in quantum information theory. In this talk, we introduce natural probability distributions for covariant quantum channels. Specifically, this is achieved through the application of 'twirling operations' on random quantum channels derived from the Stinespring representation that use Haar-distributed random isometries. We explore two types of group symmetries, namely hyperoctahedral covariance and diagonal orthogonal covariance, and analyze their properties related to quantum entanglement based on the model parameters. In particular, we discuss the threshold phenomenon for positive partial transpose and entanglement-breaking properties, comparing thresholds among different classes of random covariant channels.

This is joint work with Ion Nechita.

Ergodic states on type III_1 factors and ergodic actions

Monday
26 Feb
10:50–11:20

Stefaan Vaes
KU Leuven

I report on a joint work with Amine Marrakchi. The talk starts with a brief introduction to Tomita-Takesaki theory for von Neumann algebras, introduced in 1970 and the starting point to unravel the structure of the at that point mysterious von Neumann algebras of type III. It was proven in 1978 that if a von Neumann algebra admits a state with trivial centralizer, then it must be a type III_1 factor. The converse implication remained open. I will present a solution of this problem, proving that such ergodic states form a dense G_δ set among all faithful normal states on any III_1 factor with separable predual.

Averaging multipliers on locally compact quantum groups

Tuesday
27 Feb
11:50–12:20

Christian Voigt
University of Glasgow

After explaining some background on approximation properties, I will discuss an averaging procedure for cb multipliers on locally compact quantum groups with respect to a compact quantum subgroup. In the special case of Drinfeld doubles of discrete quantum groups this leads to a succinct description of central approximation properties. Time permitting I will also mention some related results and open problems. (Joint work with M. Daws and J. Krajczok)

Pseudodifferential operators in the noncommutative setting

Tuesday
27 Feb
09:35–10:05

Xiao Xiong
Harbin Institute of Technology

The theory of pseudo-differential operators connects partial differential operators with harmonic analysis. It is an important tool in the study of PDE and differential geometry. It has recently been studied by McDonald, Sukochev and Zanin in a C*-algebraic way, which makes it possible to extend the theory to the noncommutative setting. In this talk, I will briefly discuss recent progress of this theory in some noncommutative spaces, mainly on symbol calculus and asymptotic limit of some pseudo-differential operators. Finally I will give an application to Connes' quantum differential and integration in noncommutative geometry.

A group-theoretic approach to PPT entanglement

Tuesday
27 Feb
11:15–11:45

Sang-Gyun Youn
Seoul National University

In quantum information theory, group symmetries have played crucial roles in studying quantum states and quantum channels. There have been somewhat sporadic but lots of efforts to analyze PPT entanglement under symmetries. We suggest an abstract approach using group symmetries, which offers additional advantages on the Horodecki criterion. In this case, we need to investigate only a much smaller number of positive maps. Indeed, we apply this approach to exhibit two main applications to study PPT entanglement: (1) there is no PPT non-entanglement-breaking quantum channel generated by the identity map, transpose map, depolarizing map, and diagonalization map, (2) there is no A-BC PPT entanglement for a class tripartite invariant quantum states with unitary group symmetries. These conclusions resolve open questions raised in some recent papers. In particular, the latter conclusion provides a strong contrast to the fact that there exist PPT entangled tripartite Werner states.

Quantization of coadjoint orbits

Monday
26 Feb
10:05–10:35

Shilin Yu
Xiamen University

The coadjoint orbit method of Kirillov and Kostant suggests that irreducible unitary representations of a Lie group can be constructed as quantization of coadjoint orbits of the group. Later Vogan reformulated the orbit method in algebro-geometric language for noncompact reductive Lie groups. Namely, it is conjectured that some representations, called "unipotent representations", can be attached to vector bundles on nilpotent orbits under certain assumptions. I will sketch the construction of these representations via deformation quantization.This is based on joint work with Conan Leung and a preprint coauthored with Ivan Losev.
