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Modeling the change in the stiffness parameters of bone tissue under the influence of external loads

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Abstract. The paper considers the problem of restructuring trabecular bone tissue under external loads. The evolutionary relationships was used in Cowin statement, changes of orientation of the pores was described in terms of the fabric tensor and the volume fraction of solid bone. To estimate the elastic properties, the mechanical constants were recalculated according to the actual states of the fabric tensor. The process of bone remodeling of a rounded rectangular area under external loads was simulated; using the theory of lazy zone and physical relations, the initial value of the change in the proportion of solid bone volume is obtained. The initial state was given by a homogeneous porous state. The boundary-value problem was solved by the finite element method, and the initial problem was solved by the method of finite differences. As a result, fields were obtained for the distribution of the mechanical properties of bone tissue in the region.

1. Introduction

Bone is a complex structure with an uneven anisotropic nature and physiological adaptive function. Nowadays, in biomechanics, the actual task is to describe the evolution of bone tissue. The physiological restructuring of the bone structure occurs when new functional conditions change the load on a single bone or part of the skeleton. This includes professional restructuring, as well as restructuring caused by changes in the static and dynamic state of the skeleton in inactivity, after amputations, traumatic deformities, with ankylosis etc. [1-3]. Recently, the description of such structures using the fabric tensor has become widespread [4-7].

In general, the relationship between the elastic constants and the structure tensor contains nine functions from the invariants of the structure tensor and the density of the material [8-10]. Later, under the assumption that the eigenvalues of the fabric tensor are normalized [9-11], these relations were simplified and already contained the first and second degree of the fabric tensor. Many tasks of the musculoskeletal system require a description of the stress-strain state of bone tissue, taking into account the formation of its structure over time when changing external loads. For example, in the tasks of mechanical interaction in the joints, it is necessary to take into account the characteristics of the tissue under the joint surfaces, as well as to be able to assess changes in the structure of orthopedic interventions in the joint, since these changes can significantly affect the strength and cruelty of the bone [1-3, 11]. Moreover, analogies can be obtained in soil mechanics [12-14].

In the present work the problem of trabecular bone tissue evolution is considered. The problem of half-plane loaded by distributed pressure was considered [11]. The initial state of the tissue was assumed to be uniformly filled with round pores [11]. In the paper the evolutionary model in Cowin statement [8-10] was considered.

2. Materials and Methods

In the paper, for the relation between the mechanical orthotropic properties of the material and the fabric tensor, the following relationships were used [4-6]:

$$\begin{aligned}
\frac{1}{E_{ii}} &= \frac{1}{E_{tissue}}\left(k_1 + 2k_6 + \left(k_2 + 2k_7\right)II + 2\left(k_3 + 2k_8\right)\lambda_i + \left(2k_4 + k_5 + 4k_9\right)\lambda_i^2\right) \\
\frac{-\nu_{ij}}{E_{ii}} &= \frac{1}{E_{tissue}}\left(k_1 + k_2 II + k_3\left(\lambda_i + \lambda_j\right) + k_4\left(\lambda_i^2 + \lambda_j^2\right) + k_5\lambda_i\lambda_j\right) \\
\frac{1}{G_{ii}} &= \frac{1}{E_{tissue}}\left(k_6 + k_7 II + k_8\left(\lambda_i + \lambda_j\right) + k_9\left(\lambda_i^2 + \lambda_j^2\right)\right)
\end{aligned} \quad (1)$$

$i, j = \overline{1,3}$

E_{ii} - elastic modulus of material elasticity with pores;

E_{tissue} - elastic modulus of material elasticity without pores;

ν_{ij} - poisson ratio;

G_{ii} - shear modulus.

k_j , $j = \overline{1,9}$ - the density function of the bone matrix;

II – the second invariant of the fabric tensor;

λ_i , $i = \overline{1,3}$ - normalized eigenvalues of the fabric tensor.

To describe the evolution of structural properties we considered a simplified form. On the assumption that fabric and porosity have no direct relationship we can simplify the formula. In this case the relationship implemented indirectly thru strains. And the following relationships can be performed [4-6]:

$$\begin{aligned}
\frac{d\tilde{K}}{dt} &= h_1\left(\tilde{\varepsilon} - tr\tilde{\varepsilon}\cdot\frac{1}{3}\tilde{I} - \left(\tilde{\varepsilon}_0 - tr\tilde{\varepsilon}_0\cdot\frac{1}{3}\tilde{I}\right)\right) + \\
&+ h_2\left(tr\left(\tilde{K}\left(\tilde{\varepsilon} - \tilde{\varepsilon}_0\right)\right)\tilde{I} - \frac{3}{2}\left(\tilde{K}\left(\tilde{\varepsilon} - \tilde{\varepsilon}_0\right) + \left(\tilde{\varepsilon} - \tilde{\varepsilon}_0\right)\tilde{K}\right)\right)
\end{aligned} \quad (2)$$

when:

$\tilde{K}$ - the deviator of the fabric tensor;

e - change in the fraction of solid volume;

$\tilde{\varepsilon}$ and $\tilde{\varepsilon}^0$ - tensors of actual and initial deformations, respectively;

h_i , $i \in \overline{1,4}$ - constants that have a dimension, determined empirically so that the restructuring of bone tissue occurs over a period corresponding to the natural reality [10, 11];

$\tilde{I}$ - unitary dimension matrix.

The evolutionary relationship describing the change in the fraction of solid bone volume in the region under consideration [4-6]:

$$\frac{de}{dt} = (f_1 + f_2 e)(tr\tilde{\varepsilon} - tr\tilde{\varepsilon}_0), \vec{x} \in \bar{S}, t \geq 0 \quad (3)$$

A two-dimensional model of the bone structure that possesses the properties of an orthotropic material is considered.

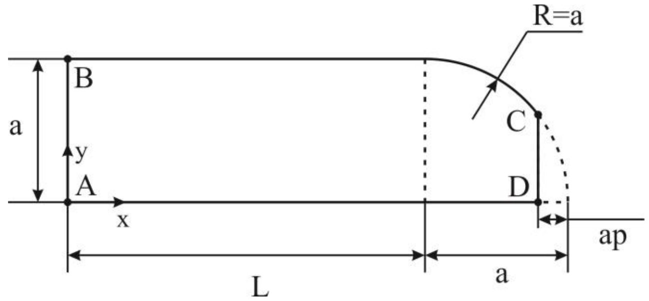

Figure 1. Computational scheme.

Here the parameters of the system are the length L, the width a and ap – the truncation fraction. In the simulation, we introduced the parameter L/a, which characterizes the relative size of the model.

Physical relationships [4-6, 8]:

$$\tilde{\sigma} = (g_1 + g_2 e) tr\tilde{\varepsilon} \cdot E + (g_3 + g_4 e)\tilde{\varepsilon} + g_5(\tilde{\varepsilon}\tilde{K} + \tilde{K}\tilde{\varepsilon}) + \quad (4)$$
$$+ g_6\left(tr(\tilde{K}\tilde{\varepsilon}) \cdot E + tr\tilde{\varepsilon} \cdot \tilde{K}\right), \vec{x} \in \bar{S}, t \geq 0,$$

where e is the change in the fraction of the solid volume of the bone, $\tilde{K}$ – the deviator of the structure tensor, and g_i – are the elastic constants.

Cauchy geometric relations:

$$\tilde{\varepsilon} = \frac{1}{2}\left(\vec{\nabla}\vec{u} + \vec{u}\vec{\nabla}\right), \vec{x} \in \bar{S}, t \geq 0 \quad (5)$$

Boundary conditions:

$$\vec{n} \cdot \tilde{\sigma} = \vec{q}, x \in S_q, t \geq 0$$
$$\vec{n} \cdot \tilde{\sigma} = 0, x \notin S_q, t \geq 0 \quad (6)$$

Initial conditions:

$$\tilde{K} = \tilde{K}_0, e = e_0, x \in \bar{S}, t = 0 \quad (7)$$

By definition, the fabric tensor is normalized so that the trace is equal to one. In the simulation it was assumed that in the initial moment of time the bone tissue is composed of uniform, round pores, which means that the deviator of the tensor structure in the initial time equal to zero: $\tilde{K}_0 = 0$. The following formal approach was used to determine the initial value of the change in the proportion of solid bone volume. It is known that bone restructuring does not occur when the deformities are equal $\varepsilon_{ij} = - 15 \cdot 10^{-4}\delta_{ij}$ the so-called lazy zone. Then, by substituting the lazy zone strain tensor and the stress tensor into the ratio (2), the initial value of the fraction solid bone volume e_0 can be determined.

Since equations (1)-(5) are a system of nonlinear algebra-differential equations, the solution of the problem was implemented numerically. In this case, the ABCD region was considered (see figure 1), which was divided into a regular grid and in which the equations were solved. In numerical simulation the following values were used for constants [4-6]: g_1 = 154.9 GPa, g_2 = 1147 GPa, g_3 = 612.9 GPa, g_4 = 4536 GPa, g_5 = 2384 GPa, g_6 = 510.8 GPa, h_1 = 0.01 1/day., h_2 = 0.02 1/day., f_1 = -2.5 1/day., f_2 = 5 1/day. In the calculations applied a unit load.

3. Results

Several numerical solutions with different parameters were carried out for the problem in order to display the most characteristic results. To determine the quality of bone structure reconstruction, the visualization of the eigenvectors of the structure tensor at fixed time moments was implemented. The solution used 300 iterations and a time step Δt=0.06 day. Since the dimension of time is a day, the step is about 1.5 hours.

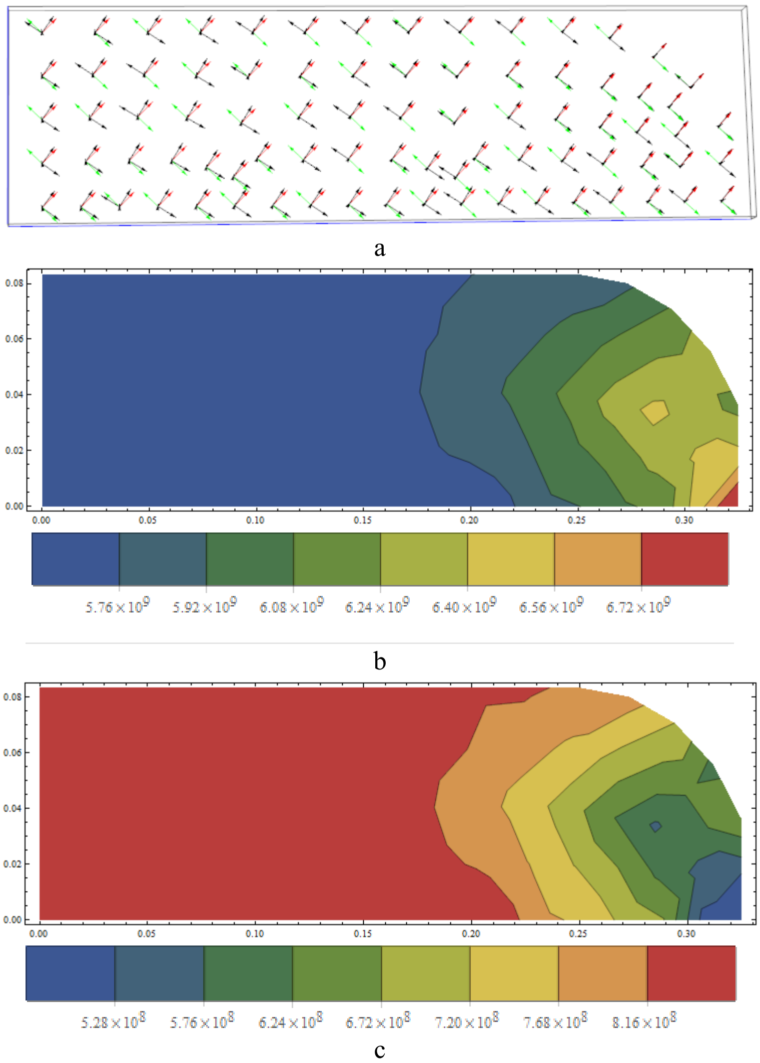

Figure 2. Results: a – eigenvectors of fabric tensor and stress tensor after 20 days; b – distribution of Young's modulus (Pa) in x direction; c - distribution of Young's modulus (Pa) in y direction.

In order to determine the convergence of the mathematical model, we will also visualize the direction of the eigenvectors of the strain tensor. The angle between the eigenvectors of the corresponding tensors will be denoted by α.

At the initial stage after the application of the load, the angle α increases sharply in the entire area under consideration, but with increasing time, a decrease in the angle near the line of application of the load can be noted, which indicates the beginning of the restructuring of the bone structure.

As a result of the reconstruction modeling, the following results were obtained within 20 days: the alignment of the eigenvectors of the structure tensor and deformation tensors is observed (see figure 2a). The results of the calculations for orthotropic properties are shown in the figure, Young's modulus in the x direction – see figure 2b, the Young's modulus in y direction – see figure 2c. There was an increase in Young's modulus in the x direction on the articular surface, in the area of application of the load. The Young's modulus in the y direction is inversely increased in the metaphysical part.

4. Discussion

The paper considers the problem of constructing a method of calculating the adaptive processes that occur in bone tissue for a given regular area. The combination of the finite element method for the boundary value problem and the finite difference method for the initial problem allowed to obtain adequate results. The calculation for the rounded rectangle is carried out, the calculation for the process of adaptation according to the given method is carried out. After the simulation of bone tissue remodeling within 20 days, the appearance of homeostasis was shown. The analysis of the results for the stiffness properties of the tissue was carried out.

5. Conclusion

The paper considers the problem of remodeling of trabecular bone tissue in a regular area under external loads. It was took into account the evolutionary relationships in the Cowin formulation, describing the change in pore orientation in terms of the fabric tensor and the proportion of solid bone volume in the considered area. The loading process of the rounded rectangle by the distributed pressure was simulated; the initial value of the change in the proportion of the solid volume of the bone was obtained using the theory of lazy zone and physical relations. The task was numerically implemented. The results obtained illustrate the convergence, for the directions of the fabric tensor, indicating that the state of homeostasis has been achieved.

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