Copy Edited by: B.P.
The Quarterly Journal of Mathematics
Quart. J. Math. 00 (2012), 1–12; doi:10.1093/qmath/has015

STRUCTURE OF ASSOCIATIVE SUBALGEBRAS OF JORDAN OPERATOR ALGEBRAS

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[Received 17 February 2012]

Abstract

We show that any order isomorphism between ordered structures of associative unital JB-subalgebras of JBW algebras is implemented naturally by a Jordan isomorphism. Consequently, JBW algebras are determined by the structure of their associative unital JB subalgebras. Furthermore, we show that in a similar way it is possible to reconstruct Jordan structure from the order structure of associative subalgebras endowed with an orthogonality relation. In case of abelian subalgebras of von Neumann algebras we show that order isomorphisms of the structure of abelian C^*-subalgebras that are well behaved with respect to the structure of two-by-two matrices over original algebra are implemented by $*$-isomorphisms. Q2

1. Introduction and preliminaries

JB algebra is a far reaching generalization of the self-adjoint part $(A, \circ)$ of a C^*-algebra $(\mathcal{A}, \cdot)$ endowed with the product

$$x \circ y = \tfrac{1}{2}(xy + yx), \quad x, y \in A.$$

Jordan product is not associative in general. Associative Jordan algebras are quite special. More precisely, basic result of spectral theory says that JB algebra $(A, \circ)$ is associative if and only if it is isomorphic to the algebra $C_0(X)$ of all continuous real-valued functions on a locally compact Hausdorff space X vanishing at infinity. Analogously, JBW algebras, i.e. JB algebras with preduals, are generalization of von Neumann algebras. The associative subalgebras of a given JB algebra are mutually overlapping and, when ordered by set-theoretic inclusion, they form a partially ordered set (poset for short). This poset can be classified as a semi-lattice: any two elements admit an infimum, which is a set theoretic intersection. If two JBW algebras are isomorphic, then their corresponding posets of associative subalgebras are order isomorphic, and so we obtain an order theoretic invariant of JBW algebras in this way. An interesting question arises whether, on the contrary, the poset of associative subalgebras determine fully the structure of JBW algebras. More precisely, in this note we shall be mainly concentrated on the following problem: let A_1 and A_2 be JBW algebras and $\mathcal{L}(A_1)$

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