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ORIGINAL PAPER

Distribution of the potential and concentration of electrons in low-temperature plasma with hollow microparticles

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Abstract Using approximation of a uniform background (the jellium model) for a condensed dispersed phase, the analytical expressions describing a spatial distribution of the potential of the electric field and electron concentration in the low-temperature plasma at equilibrium which contains hollow spherical microparticles are obtained. The influence of heating temperature of plasma on the above distributions is studied, and the dependencies of the charge on microparticle radius, the size of the microparticle cavity and the absolute temperature of plasma are calculated. It is shown that electrons can be emitted not only into the surrounding plasma but also into the cavity of the particles.

Keywords Dusty plasma · Hollow particle · Jellium model

Introduction

Low-temperature plasma which contains small particles, or so-called dusty plasma, is a complex system with variation of the dust particle's charge, mass and size within plasma space distribution and time evolution (Fortov et al. 2004; Fortov et al. 2012; Shukla and Mamun 2002; Vladimirov et al. 2005; Couedel et al. 2010). Dusty plasma can be found in interplanetary space, comet tails and Earth's atmosphere (Shukla and Mamun 2002), as well as it accompanies many technological processes such as dc and rf discharges and solid-fuel combustion products. (Thomas 2009; Valderrama et al. 2010). Dust particles may be purposely introduced to plasma or may be formed in it spontaneously.

The interest in the dusty plasma has been raised significantly during the last decades, and the quantity of publications in the period from 1981 to 2004 increased exponentially with the e-folding time of 3.9 years (Merlino 2005). Such a significant growth of publications on this subject was driven primarily by discoveries in two different areas of research which are considered as the milestone events in dusty plasma physics. In planetary science, it was discovered the spokes in Saturn's B ring (Smith et al. 1982). In applied plasma science, it was shown that the contamination of semiconductor material in plasma processing tools was due to particles grown in the plasma (Selwyn et al. 1990). The above discoveries gave an impetus to a number of other research works in both basic and applied plasma physics.

Dusty plasma is more complex as compared to normal electron–ion plasma, because it contains additional charged micron- or submicron-sized particles. This extra component of microparticles further increases the complexity of the system, so that it is also referred to as 'complex plasma'. Thus, dusty plasma is low-temperature fully or partially ionized electrically conducting gases whose constituents are electrons, ions, charged dust grains and neutral atoms. Dust grains can be massive and much more heavy than the protons, and their sizes are in the range from nanometers to millimeters. Dust grains may be metallic, dielectric, made of carbon, ice, etc., particulates.

Dusty plasma can be obtained, for example, as a result of combustion of a metal powder in the presence of some gaseous oxidizer. The product of burning contains finely divided particles of metallic oxides, whose size may vary

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from tens of nanometers to a few microns depending on combustion conditions (Zolotko et al. 1996). With temperature of combustion products in the range of 1000–3000 °K, the condensed particles become charged as a result of electron emission from their surface; therefore, the gaseous phase contains free electrons as well (Song et al. 2016; Delzanno and Tang 2014; Samarian et al. 2001; D'yachkov et al. 2008; Khrapak et al. 2007; Fomenko 1981; Fairushin et al. 2014; Dautov et al. 2014). However, if there are no easily ionized particles in the gaseous phase, gas ionization can be impeded even at such temperatures. The resulting mixture, which consists of neutral atoms, positively charged microparticles and free electrons, is described as dusty plasma.

Dusty plasma composed of hollow particles has not been a subject of detailed studies so far. At the same time, the modern-day industry makes increasing use of powders that consist of hollow particles. For instance, when solid fuels are burned at thermal electric power stations, aluminosilicate hollow microspheres are formed which are used as fillers in the production of composite materials with thermal insulation and sound-proofing properties (Ma et al. 2007). Hollow microparticles are also a basis for catalysts and adsorbents (Solonenko et al. 2011). The advantage of use of powders consisting of hollow particles in the process of plasma spraying for preparation of thermo-resistant and wear-resistant coatings is that it results in more intensive heating and uniform temperature distribution across the particle volume, reduction in the quantity of unmelted particles in the plasma spray as well (Solonenko et al. 2011).

In order to understand the properties of dusty plasmas based on hollow microparticles and how they affect various aspects of technology as well as regions in space and in the earth's atmosphere, it is necessary to undertake both theoretical and laboratory investigations of dust systems composed of hollow particles. The current work theoretically studies the influence of heating temperature of plasma on the distribution and magnitude of microparticles' charge depending on their radius, size of their cavity and absolute temperature of plasma. Particularly, we show that electrons are emitted not only into the surrounding plasma but also into the hollowness of a particle.

Materials and methods

The content of dust particles in plasma makes it possible to use models from the physics and chemistry of condensed states to describe the structures of these particles. One of the models is the so-called jellium model (Ekardt 1984; Ivanov et al. 1996; Smirnov and Krainov 1999). In this model, interacting electrons are uniformly distributed in a positively charged solid (i.e., atomic nuclei). An ionized particle's core is considered as a continuous homogeneous positive background, and the electron density is a uniform quantity as well in that space. This model allows one to focus on the effects in solids that occur due to the quantum nature of electrons and their mutual repulsive interactions (due to the same charge) without explicit introduction of the atomic lattice and structure making up a real material. The jellium model is widely used for describing electronic properties of metals and semiconductors as well as their clusters and particles (Ekardt 1984; Ivanov et al. 1996; Smirnov and Krainov 1999).

Results and discussions

Let us consider dusty plasma which consists of hollow spherical particles of matter, free electrons and neutral atoms of surrounding gas. Let us assume that the particle cavity contains the same gas as the surrounding one. Let us assume that there are no bound charges inside the particle, i.e., the medium is not polarized and the relative dielectric constant ε is equal to 1. Let absolute temperature T of all the components of the system be equal (i.e., consider that the dusty plasma is in equilibrium) (Vishnyakov and Dragan 2006; Vishnyakov et al. 2007; Vishnyakov 2012; Smirnov et al. 2007; Vaverka et al. 2014; Krauz et al. 2010). When the gas microparticles emit electrons, an energy barrier is formed at their surfaces to prevent further emission. This barrier also determines the work function of electrons to escape from the particle into the environment (Ekardt 1984). This results in a statistical charge balance between the particles and the surrounding plasma (Ashcroft and Mermin 1976; Artsimovich and Sagdeyev 1979; Landau and Lifshits 2010). Let the distance between the centers of two neighboring particles of radius R be equal to $2\,l$. R_1 is the radius of the particle's cavity, whereas r is the current coordinate, measured from the center of a particle (Fig. 1).

Let us assume that the particles consist of semiconductor material, and the electron gas in the conductive band of this material is non-degenerate. Let the concentration of ions (the density of positively charged background) in the region $R_1 \le r \le R$ be equal to n_i. Let us find the distribution of potential $\varphi(r)$ and concentration of electrons $n_e(r)$ in the region $0 \le r \le l$ under the following conditions:

$$n_i(r) = \begin{cases} 0, & \text{when } r < R_1 \text{ and } r > R \\ \text{Const}, & \text{when } R_1 \le r \le R \end{cases} \quad (1)$$

$$\varphi(0) = 0, \varphi'(0) = 0 \quad (2)$$

The electric field is described by the divergence theorem

$$\oiint_s \varepsilon\varepsilon_0 E_n \mathrm{d}S = \iiint_v (n_i - n_e) q \mathrm{d}V, , \quad (3)$$

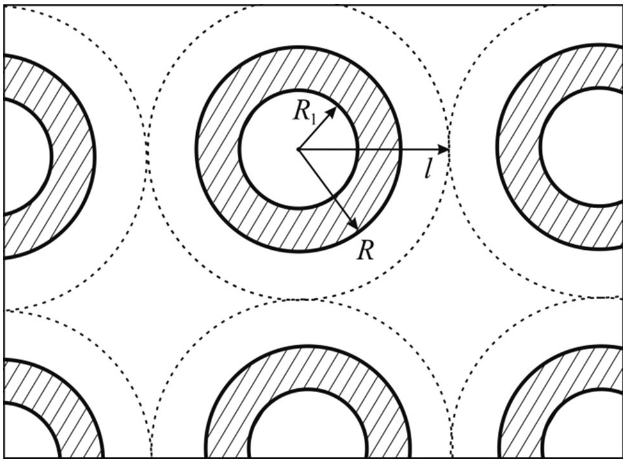

Fig. 1 Model presentation of the problem

where ε is the relative dielectric constant; ε_0 the vacuum permittivity; S an arbitrary closed surface; E_n a projection of the vector of the electric field on the outward normal to the surface S; V the volume spanned by the surface S; and q the absolute value of the electron charge.

Taking into account the Boltzmann distribution of the electron gas, i.e.,

$$n_e = n_{e0} e^{\frac{q\phi}{kT}} \tag{4}$$

and a spherical symmetry, we obtain from Eq. (3) the Poisson–Boltzmann equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \frac{q}{\varepsilon\varepsilon_0}\left(n_{e0}e^{\frac{q\phi}{kT}} - n_i\right), \tag{5}$$

where k is the Boltzmann's constant and n_{e0} the concentration of electrons at $\varphi = 0$. The value of n_{e0} is established on condition $\varphi'(l) = 0$, which determines the balance of full electric charge in the volume of a single dust particle (so-called Wigner–Seitz cell) (D'yachkov et al. 2008).

By introducing dimensionless values $x = \frac{r}{R}$, $\psi = \frac{q\phi}{kT}$, $\bar{n}_i = \frac{n_i}{n_{e0}}$, $a^2 = \frac{q^2R^2n_{e0}}{kT\varepsilon\varepsilon_0}$, we obtain from Eq. (5),

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{d\psi}{dr}\right) - a^2\left(e^{\psi} - \bar{n}_i\right) = 0. \tag{6}$$

Let us examine first the case of $|\psi| << 1$. We can decompose the value e^{ψ} into a series and restrict ourselves by the first two components

$$e^{\psi} = 1 + \psi$$

The exact solution for this case will be:

$$\psi = \frac{e^{ax} - e^{-ax}}{2ax} - 1, \quad \text{when} \quad 0 \le x < x_1 \tag{7}$$

$$\psi = \bar{n}_i - 1 + \frac{C_3e^{ax} - C_4e^{-ax}}{x}, \quad \text{when} \quad x_1 \le x \le 1 \tag{8}$$

$$\psi = \frac{C_5e^{ax} - C_6e^{-ax}}{x}, \quad \text{when} \quad 1 < x \le \lambda \tag{9}$$

In this case,

$$x_1 = \frac{R_1}{R}, \; \lambda = \frac{R}{l} \tag{10}$$

$$C_3 = \frac{1 - \bar{n}_ie^{-ax_1}(1 + ax_1)}{2a} \tag{11}$$

$$C_4 = \frac{\bar{n}_ie^{ax_1}(1 - ax_1) - 1}{2a} \tag{12}$$

$$C_5 = \frac{1 - \bar{n}_ie^{-ax_1}(1 + ax_1) + (a+1)\bar{n}_ie^{-a}}{2a} \tag{13}$$

$$C_6 = \frac{\bar{n}_ie^{ax_1}(1 - ax_1) - 1 + (a-1)\bar{n}_ie^{a}}{2a} \tag{14}$$

$$\bar{n}_i = \frac{e^{-2a\lambda}(a\lambda + 1) + a\lambda - 1}{e^{-2a\lambda}(a\lambda + 1)(e^{ax_1}(1 - ax_1) + e^a(a-1)) + (a\lambda - 1)(e^{-ax_1}(ax_1 + 1) + e^{-a}(a+1))} \tag{15}$$

From Eqs. (7)–(15), taking into consideration the Boltzmann distribution, we can obtain the formulas for distribution of concentration of electrons:

$$n_e = n_{e0}\exp\left[\frac{e^{ax} - e^{-ax}}{2ax} - 1\right], \quad \text{when } 0 \le x < x_1, \tag{16}$$

$$n_e = n_{e0}\exp\left[\bar{n}_i - 1 + \frac{C_3e^{ax} - C_4e^{-ax}}{x}\right], \quad \text{when } x_1 \le x \le 1 \tag{17}$$

$$n_e = n_{e0}\exp\left[\frac{C_5e^{ax} - C_6e^{-ax}}{x} - 1\right], \quad \text{when } 1 < x \le \lambda \tag{18}$$

From the above formulas, it is clear that the distributions of the dimensionless concentration of electrons $\overline{n_e} = \frac{n_e}{n_{e0}}$ and

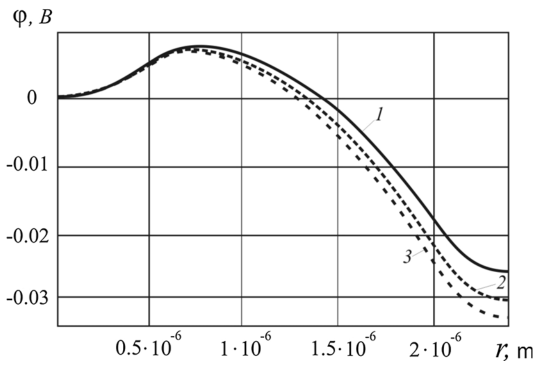

Fig. 2 Distribution of potential for various temperatures in the region $0 \le r \le l$ *at* $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2 \times 10^{-6}$ m $l = 2.4 \times 10^{-6}$ m (*1*—for $T = 1000$ K, *2*—for $T = 1500$ K and *3*—for $T = 2000$ K)

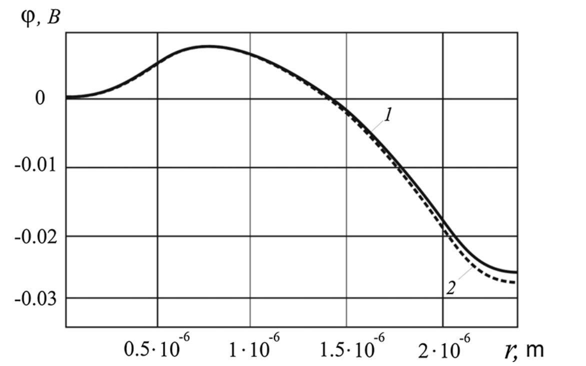

Fig. 3 Distribution of the potential in the region $0 \le r \le l$ *at* $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2 \times 10^{-6}$ m, $l = 2.4 \times 10^{-6}$ m and $T = 1000$ K (*1* is the result of analytical solution, *2* the computational solution)

dimensionless potential ψ are the functions of dimensionless values a, λ, x and x_1, i.e.,

$$\psi = f_1(a, \lambda, x_1, x),\ \overline{n_e} = f_2(a, \lambda, x_1, x)$$

Thus, the values

$$a = qR\sqrt{\frac{n_{e0}}{\mathrm{kT}\varepsilon\varepsilon_0}}, \quad \lambda = \frac{l}{R}, \quad x_1 = \frac{R_1}{R}, \quad x = \frac{r}{R}$$

are the characteristic values of these functions as well as criteria of similarity for this problem. Consequently, at $a = \text{const}$, $\lambda = \text{const}$ and $x_1 = \text{const}$, the generalized distributions of $\psi(x)$ and $\overline{n_e}(x)$ coincide.

According to Fig. 2, the potential rises slowly within the cavity of the particle with increasing the radius vector from the particle center, *r*. Then, as electrons approach the internal surface, the potential rises faster, which is explained by high value of the electron gradient. In the area $R_1 \le r \le R$, the potential slowly increases, reaches its maximum and then starts to decrease. The decrease in the potential with increasing *r* is most quick near the external surface. This leads to high strength of the electric field. From Eq. (8), it is possible to find a formula for calculation of this strength on the external surface of a hollow microparticle, i.e., in the point $r = R$:

$$E(R) = -\frac{\mathrm{kT}}{qR}\psi'_x(l) \tag{19}$$

This formula yields $E(R) = 4.2 \times 10^4$ V/m at $T = 1000$ K, $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2 \times 10^{-6}$ m and $l = 24 \times 10^{-6}$ m. In the region $R < r \le l$ with increasing *r*, the potential decreases initially faster and then slower to value $\varphi(l)$.

Calculations show that with high *l* the condition $|\psi| \ll 1$ is not valid. In this case, Eq. (6) was solved computationally by the Runge–Kutta algorithm.

Curve *1* in Fig. 3 is plotted according to formulas (7)–(9), and curve *2* is plotted according to computational solution of differential Eq. (6). As it is seen from the comparison of these curves, the linearization of Eq. (6) leads to overestimated value of the potential at high *r*. Calculations of the potential distribution showed that it is possible to overcome the potential barrier for the electron close the surface of the particle. This surface determines the work function of electron to leave the particle into the surrounding gas. It was obtained that at the above set of parameters the height of the potential barrier does not exceed several tens of meV. Such a value is too small as compared to the work function of real substances. Therefore, the above estimates allow us to suggest an appropriate qualitative description of the processes of electron emission from the dust particles.

Figure 4 presents the results of calculations of the distribution of electron concentration according to formulas (16)–(18). With increasing *r* in the region $r \approx R$, the value n_e decreases abruptly. With further increasing *r* away from the particle, the rate of change in n_e decreases.

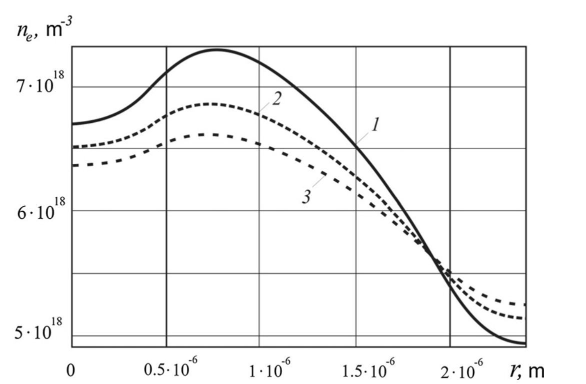

Fig. 4 Distribution of concentration of electrons for different temperatures in the region $0 \le r \le l$ at $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2 \times 10^{-6}$ m and $l = 24 \times 10^{-6}$ m (*1*—for $T = 1000$ K, *2*—for $T = 1500$ K and *3*—for $T = 2000$ K)

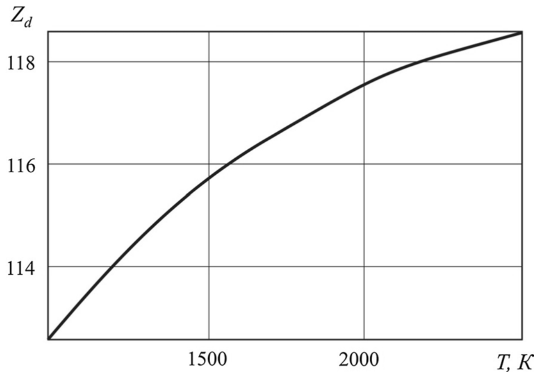

Fig. 5 Dependence of the microparticle's charge on the absolute temperature at $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2{\times}10^{-6}$ m and $l = 24 \times 10^{-6}$ m

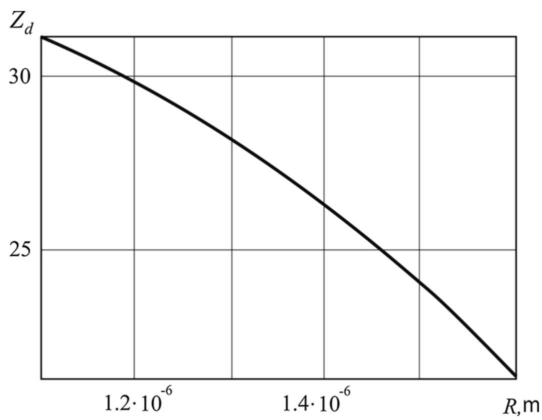

Fig. 6 Dependence of the microparticle's charge on its outer radius at $n_i = 10^{19}$ m^{-3}, $R_1 = 5 \times 10^{-7}$ m, $R = 2 \times 10^{-6}$ m and $l = 2.4 \times 10^{-6}$ m

Comparison of the curves in Fig. 4 shows that at high temperatures the concentration of electrons decreases faster upon increase in r inside the particle. However, the concentration of electrons outside of the particle increases as temperature rises. Figure 4 shows that the major part of charge is located near the surface of the particle, i.e., a dust particle is surrounded by a dense electron cloud.

In order to measure the charge of a microparticle in electron charge units Z_d, let us write the divergence theorem for the case $r = R{:}E(R)4\pi R^2 = \frac{Z_d q}{\varepsilon\varepsilon_0}$, from where $Z_d = \frac{4\pi R^2 \varepsilon\varepsilon_0}{q} E(R)$,where $E(R)$ is defined according to formula (19). In fact, the value Z_d determines the value of an electric charge inside the sphere of radius R. As it is expected, Z_d increases with increasing T (Fig. 5).

Figure 6 shows the dependence of Z_d on R on condition that all the microparticles contain the same quantity of the substance, i.e.,

$$n_d\left(R^3 - R_1^3\right) = \text{Const},$$

where n_d is concentration of microparticles. Figure 6 shows that the value of Z_d decreases with increase in the outer radius R.

This result is explained by the fact that as the outer radius increases, the inner radius R_1 increases as well. Therefore, the quantity of electrons that fills up the cavity of a particle increases, and the quantity of electrons leaving the particle into the region $R < r \leq l$ decreases.

Conclusion

In this work, the ratios that can be applied for qualitative description of the major properties of dusty plasma at equilibrium which contains hollow spherical particles have been obtained. The obtained results allow us to understand the behavior of the potential, the concentration of electrons near the surface of a particle and the charge value of the dusty particles as a function of their radius and temperature. The obtained knowledge is not the only academic, but it can also be used for application purposes, such as design of processes of plasma spray coating using powder materials.

It is worth noting that there are further ways to improve the proposed model. These ways include account of polarization of atoms of the particle substance, which results in the emergence of a surface charge; that means that the relative dielectric constant of the particles' material can be different from 1. The other way includes a comparison of the above theoretical results with experimental ones for a specific substance taking into account an increase in the steady-state density of free electrons of this substance during increasing temperature. Estimate of the period of reaching a statistical equilibrium and determination of the lower level of concentration of the dust particles when both the electron gas and the gas of the environment are at equilibrium is also a future work to be done.

Finally, it should be noted that introduction of hollow particles into the dusty plasma enriches its properties and the proposed model enables us to better understand potential application of the dusty plasma in various technological processes.

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