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Analytical Calculation of Distributions of the Electron Density and the Concentration of Impurity Ions in a Thermal Dusty Plasma using the Jellium Model for Condensed Particles

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Abstract—Self-consistent spatial distributions of the electron density in the entire volume of condensed-matter particles and the surrounding plasma, as well as distributions of the concentration of ions of easily ionized impurity atoms, are obtained using the jellium model to describe particles. It is established that electron emission from condensed particles in a thermal dusty plasma containing an impurity of an easily ionized element may weaken with an increase in temperature. The electron emission from particles is shown to increase with a decrease in their radius at a constant temperature. A plasma region with violated ionization equilibrium is found to form near the surface of condensed particles.

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A system consisting of a partially ionized gas and micrometer-sized particles of condensed matter is usually referred to as “dusty plasma” or “plasma with a condensed dispersed phase” (CDP) [1]. One form of this plasma is a thermal plasma with microparticles, which are either introduced or formed therein [2–6]. Such system is formed, for example, during the combustion of metal particles, which is accompanied by the formation of metal oxides in the form of micrometer-sized particles [6, 7]. This is due to the fact that the condensed and gas media are thermally ionized at the combustion temperature, which affects all stages of the formation and growth of condensed particles [7].

Investigations of a thermal plasma containing charged condensed-matter microparticles are also urgent, because it is formed when preparing functional coatings by plasma deposition [8]. It should be noted that, since a thermal dusty plasma (TDP) is a dispersed system, the variety of its properties is due to processes occurring near the surface of condensed particles. Thus, the methods of surface physics should be applied to describe this system. To date, many results describing the electronic properties of a material surface have been obtained by researchers in the field of surface physics [9–15]. The size dependences of various parameters and the environmental influence on the surface properties were also considered. For example, a decrease in the work function was analyzed for metals coated with dielectric diamond-like, polymer, and oxide films [9–11]. When considering the problem of describing the distribution of the electric-field potential in condensed medium and near its surface, the jellium model is widely used; according to this model, the ion core of the condensed medium is considered as a continuous uniform positive background and conduction electrons as an electron gas in this positive background. This problem has been analyzed in detail, for example, in [9–14]. As applied to small particles, this problem was considered in [16–19], where the jellium model was used to describe small particles as well.

The problem of determining the distribution of the electric-field potential is also of great importance for the theoretical description of TDP. Some studies [3–5] devoted to the emission charging of dust particles in gas and plasma yielded calculation data on the distribution of the potential and electron density in the space between particles. However, the question of distribution of the potential and electron density specifically in particles as applied to dust plasma has not been considered, and the particle charge is determined from the balance equation using the Richardson–Dushman formula, in which the work function is assumed to depend on only the particle material. However, the work function also depends on the particle radius and temperature, which was shown, in particular, in [20].

In this Letter, we solve the problem of combined description of the distribution of the electric-field potential for regions inside dust particles and the surrounding plasma. We consider semiconductor particles; in this case, the positive-background density is determined by the concentration of atoms, electrons of which can freely move inside the particle. The temperatures of the components of the dust particles–plasma system are assumed to be equal, which is implemented in some cases at atmospheric or higher pressures [2].

Let us assume that the electron gas in a particle (as in the surrounding plasma) has a concentration below the degeneracy threshold; therefore, we can apply the Boltzmann distribution:

$$n_e = n_{e0} e^{\frac{q\varphi}{kT}}, \tag{1}$$

where n_e is the electron density, q is the absolute value of the elementary charge, φ is the electric-field potential, n_{e0} is the electron density at $\varphi = 0$, k is the Boltzmann constant, and T is the absolute equilibrium temperature of all system components. In this Letter, we consider the range of variation in T from 2000 to 2500 K, which is typical of TDP [2–6]. The calculations were performed with the following values of the density of the particle positive background and the relative permittivity of the particle material: $n_i^{(1)} = 10^{21}$ m^{-3} and $\varepsilon = 10$, respectively.

When the gas around the dust particles contains no easily ionized impurities, its ionization can be neglected in the temperature range under consideration. We assume here that the gas contains an impurity of an easily ionized alkaline metal. To determine the charged-component concentration in the plasma around dust particles, we use the well-known Saha equation

$$\frac{n_i^{(2)} n_e}{n_a} = \frac{2z_i}{z_a}\left(\frac{2\pi m_e kT}{h^2}\right)^{\frac{3}{2}} e^{\frac{q\varphi_u}{kT}},$$

where $n_i^{(2)}$ and n_a are the concentrations of impurity ions and atoms, respectively; n_e is the electron concentration (electron density); z_i and z_a are the partition functions of impurity ions and atoms, respectively (the values corresponding to potassium are used in further calculations); m_e is the electron mass; h is the Planck's constant; and φ_u is the impurity-atom ionization potential.

In the temperature range under consideration, particles emit electrons into the surrounding plasma and acquire some positive charge. Thus, an electric field is formed around a dust particle, which affects the spatial distribution of the electron and ion concentrations near the particle surface. For simplicity, particles are assumed to be spherical with radius R and uniformly distributed in space with distance $2l$ between the centers.

It is known that the electric-field potential and concentration of electric charges are related by the Poisson equation. In our case, where the electron gas in the entire TDP volume is nondegenerate, this equation will be the Poisson–Boltzmann equation. Taking into account that the problem under consideration is spherically symmetric and impurity ions in the plasma are described by the Boltzmann distribution, we finally obtain

$$\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = \frac{q}{\varepsilon_0}\left(\frac{n_{e0} e^{\frac{q\varphi}{kT}}}{(\varepsilon - 1)\theta(R - r) + 1} - \frac{n_i^{(1)}}{\varepsilon}\theta(R - r) - n_{i0}^{(2)} e^{-\frac{q\varphi}{kT}}\theta(r - R)\right), \tag{2}$$

where r is the coordinate counted from the particle center, ε_0 is the permittivity of free space, $\theta(r)$ is the Heaviside function, and $n_{i0}^{(2)}$ is the impurity ion concentration at $\varphi = 0$. The potential zero corresponds to the particle surface (i.e., $\varphi(R) = 0$). The problem is symmetric with respect to the particle center; therefore, $\varphi'(0) = 0$. The condition of equality of the total charge in the volume per particle to zero must also be applied (i.e., $\varphi'(l) = 0$). In the case of $|q\varphi| \ll kT$, the exponential factors on the right-hand side of Eq. (2) can be expanded in series and we can restrict ourselves to the first two terms; this equation can then be solved analytically. Below, we will consider only the solutions that well satisfy this condition. The solution was performed in two steps: first, the region $0 \le r < R$ is considered and, then, taking into account the condition of continuity of the electron density and electric field on the particle surface, the potential distribution is found in the region $R \le r \le l$. The analytical expressions describing the potential distribution can be written in the form

$$\psi(x) = (\bar{n}_i^{(1)} - 1)\left(\frac{e^{-ax} - e^{ax}}{x(e^a - e^{-a})} + 1\right) \quad \text{for} \quad 0 \le x < 1,$$

$$\psi(x) = \frac{1}{\bar{n}_{i0}^{(2)} + 1}\left(\frac{C_2 e^{bx} + C_3 e^{-bx}}{x} + \bar{n}_{i0}^{(2)} - 1\right) \quad \text{for} \quad 1 \le x \le \lambda.$$

Here,

$$\psi = \frac{q\varphi}{kT}; \quad \bar{n}_i^{(1)} = \frac{n_i^{(1)}}{n_{e0}}; \quad x = \frac{r}{R}; \quad a = qR\sqrt{\frac{n_{e0}}{\varepsilon\varepsilon_0 kT}};$$

$$\bar{n}_{i0}^{(2)} = \frac{An}{n_{e0}^2 + An_{e0}}; \quad A = \frac{2z_i}{z_a}\left(\frac{2\pi m_e kT}{h^2}\right)^{\frac{3}{2}} e^{\frac{q\varphi_u}{kT}};$$

$n = n_a + \bar{n}_{i0}^{(2)}$ and n_a are the impurity atomic concentrations before and after ionization, respectively; $b = a\sqrt{\varepsilon(1 + \bar{n}_{i0}^{(2)})}$; and $\lambda = \frac{l}{R}$. In addition,

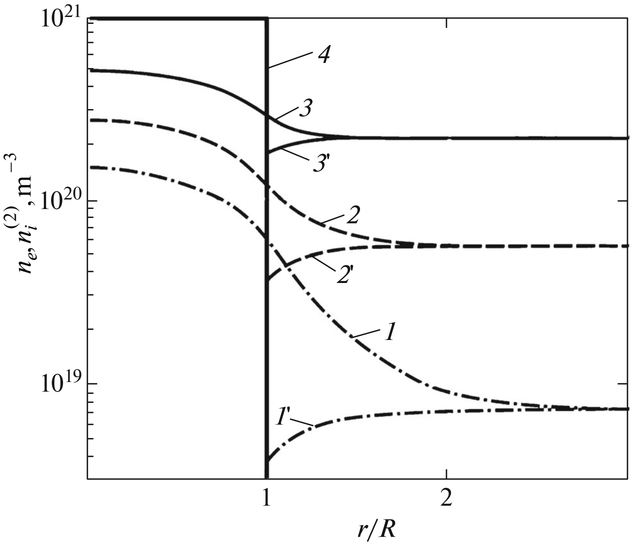

Fig. 1. Distributions of (*1*, *2*, *3*) electron density and (*1*', *2*', *3*') impurity ion concentration at $n = 10^{23}$ m^{-3}, $R = 10^{-6}$ m, $l = 3R$, and temperatures T = (*1*, *1*') 2000, (*2*, *2*') 2250, and (*3*, *3*') 2500 K; curve *4* corresponds to $n_i^{(1)}$.

$$\overline{n}_i^{(1)} = 1 - C_1(e^a - e^{-a});$$

$$C_1 = \frac{C_2 - (b+1)(1 - \overline{n}_{i0}^{(2)})}{(\overline{n}_{i0}^{(2)} + 1)(e^a(a-1) + e^{-a}(a+1))};$$

$$C_2 = \frac{(b\lambda + 1)(1 - \overline{n}_{i0}^{(2)})e^{b(1-2\lambda)}}{(b\lambda + 1)e^{2(1-\lambda)} + b\lambda - 1};$$

$$C_3 = e^b(1 - \overline{n}_{i0}^{(2)} - C_2 e^b).$$

With the potential distribution known, the distributions of the electron density and impurity ion concentration can easily be determined from formula (1). Figure 1 shows the distributions of the electron and ion concentrations in the region $0 \le r \le l$.

It can be seen in Fig. 1 that the electron density increases with an increase in temperature in the entire region from the particle center to $r = l$. The growth of the electron density in the plasma is obviously due to the increase in its degree of ionization upon heating. At the same time, the electron density inside the particle should decrease with an increase in temperature because one would expect an increase in the electron emission in this case (as in the case in which the degree of ionization of the gas around the particle is negligible [21]). In our case, the release of electrons from particles is hindered by the growing concentration of plasma electrons, which arise due to the ionization of impurity atoms. It can be seen that the free-electron concentration in plasma increases with a decrease in temperature mainly because of the electron emission from condensed particles, and the degree of ionization of impurity atoms is rather low (curves *1* and *1*').

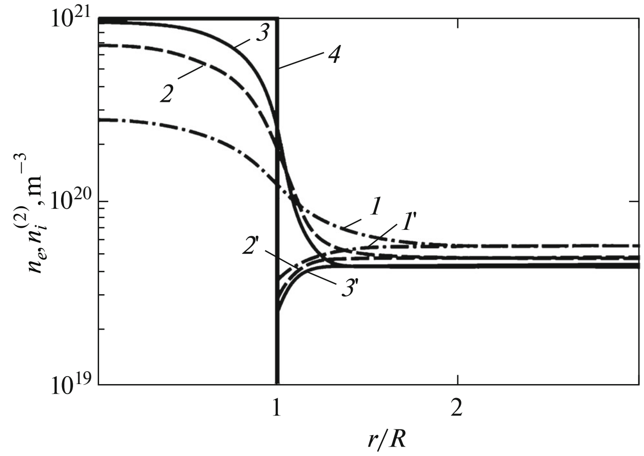

Fig. 2. Distributions of (*1*, *2*, *3*) electron density and (*1*', *2*', *3*') impurity ion concentration at $n = 10^{23}$ m^{-3}, T = 2250 K, $l = 3R$, and particle radii R = (*1*, *1*') 10^{-6}, (*2*, *2*') 2×10^{-6}, and (*3*, *3*') 3×10^{-6} m; curve *4* corresponds to $n_i^{(1)}$.

A change in the character of the spatial electron-density distribution depending on the particle radius is also observed (Fig. 2). The emission of electrons from the particle into the surrounding plasma is intensified with a decrease in the particle size. The reason is the increase in the specific surface with a decrease in the particle radius. Thus, smaller particles emit electrons into the surrounding plasma more efficiently.

It should be noted that the ionization equilibrium is violated near the particle surface. At large distances from the particle, the ion and electron concentrations become equal and the plasma is quasi-neutral. The violation of the ionization equilibrium is more pronounced at lower plasma temperatures and smaller particle radii.

Note that the problem solved in this Letter is similar to that arising when determining the electron concentration distribution at the point of contact between two n-type semiconductors with different free carrier concentrations (in this case, electrons) (see, for example, [22]). In essence, the difference is that, in our case (when considering the contact of the equilibrium plasma with semiconductor), positive-charge carriers from the side of the plasma are mobile and affected by the electric field induced by the charge redistribution.

In conclusion, it should be noted that, by choosing the parameters of thermal plasma around the condensed particles, one can affect the value and distribution of the electron density in space between dust particles and, correspondingly, the dusty-plasma electrical conductivity as a whole. This is important for a number of applications, such as, for example, formation of coatings by plasma deposition and plasmochemical synthesis of dispersed materials.

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REFERENCES

1. V. E. Fortov, A. G. Khrapak, and I. T. Yakubov, *The Physics of Non-Ideal Plasma, The School-Book* (Fizmatlit, Moscow, 2004) [in Russian].
2. V. I. Vishnyakov, Phys. Rev. E **85**, 026402 (2012).
3. V. E. Fortov, V. S. Filinov, A. P. Nefedov, et al., J. Exp. Theor. Phys. **84**, 489 (1997).
4. L. G. D'yachkov, A. G. Khrapak, and S. A. Khrapak, J. Exp. Theor. Phys. **106**, 166 (2008).
5. S. A. Khrapak, G. E. Morfill, V. E. Fortov, et al., Phys. Rev. Lett. **99**, 055003 (2007).
6. G. S. Dragan, J. Exp. Theor. Phys. **98**, 503 (2004).
7. A. N. Zolotko, N. I. Poletaev, and Ya. I. Vovchuk, Combust. Explos., Shock Waves **51**, 252 (2015).
8. A. F. Puzryakov, *Theoretical Principles of Plasma Spraying Technology* (Mosk. Gos. Tekh. Univ. im. N.E. Baumana, Moscow, 2003) [in Russian].
9. Yu. M. Yumaguzin, V. M. Kornilov, and A. N. Lachinov, J. Exp. Theor. Phys. **103**, 264 (2006).
10. K. P. Loshitskaya, V. A. Sozaev, and R. A. Chernyshova, Poverkhnost', No. 9, 104 (2005).
11. A. N. Smogunov, L. I. Kurkina, and O. V. Farberovich, Phys. Solid State **42**, 1898 (2000).
12. P. K. Korotkov, V. A. Sozaev, R. B. Tkhakakhov, and Z. A. Uyanaeva, Bull. Russ. Acad. Sci.: Phys. **73**, 982 (2009).
13. M. V. Mamonova, V. V. Prudnikov, and I. A. Prudnikova, *The Physics of the Surface. Theoretical Models and Experimental Method*s (Fizmatlit, Moscow, 2011) [in Russian].
14. V. I. Roldugin, *Physical Chemistry of the Surface* (Intellekt, Dolgoprudnyi, 2011) [in Russian].
15. M. B. Partenskii, Sov. Phys. Usp. **22**, 330 (1979).
16. M. B. Partenskii, Poverkhnost', No. 10, 15 (1982).
17. M. B. Smirnov and V. P. Krainov, J. Exp. Theor. Phys. **88**, 1102 (1999).
18. W. Ekardt, Phys. Rev. B **29**, 1558 (1984).
19. V. K. Ivanov and A. N. Ipatov, J. Exp. Theor. Phys. **82**, 485 (1996).
20. B. M. Smirnov, Phys. Usp. **43**, 453 (2000).
21. G. Dautov, I. Fayrushin, I. Dautov, and N. Kashapov, J. Phys.: Conf. Ser. **479**, 012014 (2013).
22. V. L. Bonch-Bruevich and S. G. Kalashnikov, *Physics of Semiconductors* (Nauka, Moscow, 1977; VEB, Berlin, 1982).

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