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ABSTRACTS

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Работа выполнена при частичной поддержке РФФИ, проект 99-01-00355.

BLOCK PROJECTION OPERATOR ON NORMED IDEAL SPACES OF MEASURABLE OPERATORS AND ORDER ANALOG OF BANACH-SAKS PROPERTY

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Let $\mathcal{M}$ be a von Neumann algebra of operators on Hilbert space $\mathcal{H}$, $\mathcal{M}^{\rm pr}$ be the lattice of all projections of $\mathcal{M}$. Let $S(\mathcal{M})$ be the $*$-algebra of measurable operators on $\mathcal{H}$. Let τ be a semifinite normal faithful trace on $\mathcal{M}$ and let $L_1(\mathcal{M},\tau)$ be Banach space of of all τ-integrable operators from $S(\mathcal{M})$ with the norm $\|x\|_1 = \tau(|x|)$, here $|x| = (x^*x)^{1/2}$.

V. Chilin, A. Krygin and F. Sukochev (1992) showed that the block projection operator

$$\Phi(x) = \sum_{k=1}^{\infty} p_k x p_k, \quad \{p_k\}_{k=1}^{\infty} \subset \mathcal{M}^{\rm pr},\ p_k p_m = 0\ (k \neq m) \qquad (1)$$

is a positive linear contraction on $\mathcal{M}$ and on $L_1(\mathcal{M},\tau)$. This fact was used for characterization of extreme points of convex fully symmetric subsets in $L_1(\mathcal{M},\tau) + \mathcal{M}$. The interpolation theory of linear operators implies that formula (1) defines positive linear continuous operator $\Phi\colon X \to X$ for all Banach symmetric subspaces $X \subset S(\mathcal{M})$, interpolational between $L_1(\mathcal{M},\tau)$ and $\mathcal{M}$.

We show that formula (1) defines positive linear continuous operator $\Phi\colon X \to X$ with $\|\Phi\|_{X^h \to X^h} \leq 1$ for the wider class of subspaces $X \subset S(\mathcal{M})$ which meets properties (A) and (B). Our method is new even for the algebra $\mathcal{M} = \mathcal{B}(\mathcal{H})$, endowed with the canonical trace $\tau = {\rm tr}$. The operator Φ in this case was considered in book of I. Gokhberg and M. Krein (1965).

We also present the "order"analog of Banach-Saks property for $L_1(\mathcal{M},\tau)$ and for noncommutative Orlich spaces $L_f(\mathcal{M},\tau)$, cf. A. Bikchentaev, (1990; 1992).

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