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Theory

Superfluidity of Heated Fermi Systems in the Static Fluctuation Approximation

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Abstract—Superfluidity properties of heated finite Fermi systems are studied in the static fluctuation approximation, which is an original method. This method relies on a single and controlled approximation, which permits taking correctly into account quasiparticle correlations and thereby going beyond the independent-quasiparticle model. A closed self-consistent set of equations for calculating correlation functions at finite temperature is obtained for a finite Fermi system described by the Bardeen–Cooper–Schrieffer Hamiltonian. An equation for the energy gap is found with allowance for fluctuation effects. It is shown that the phase transition to the supefluid state is smeared upon the inclusion of fluctuations.

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1. INTRODUCTION

In recent years, the growth of variety of systems formed by a finite number of fermions and created and studied under laboratory conditions was among the factors that quickened interest in such systems. In addition to nuclei known for a long time, objects of this type that are being vigorously studied at present include quantum dots and clusters containing a finite number of atoms or molecules. At least some of the aforementioned Fermi systems are long-lived even in highly excited states, with the result that the evolution of their properties versus temperature can be studied experimentally. Available information about the behavior of so-called heated nuclei (those whose excitation energy is about several MeV units or even higher) is quite extensive. It includes information about changes in their shape and in collective vibrations.

The main problem in theoretically studying highly excited systems stems from an extremely high level density, in which case realistic calculations on the basis of a microcanonical ensemble are impossible, so that one has to resort to a statistical description. Therefore, it is of importance to construct an efficient statistical theory that would describe a broad range of phenomena in heated finite Fermi systems.

In studying the properties of highly excited finite Fermi systems, one usually generalizes methods and approximations developed for weakly excited systems (at $T = 0$). However, many of the approximations used give no way to take correctly into account various correlation effects, which could play an important role in explaining observed statistical properties.

By employing the concept of temperature, one can generalize a number of methods developed for studying weakly excited Fermi systems to the case of highly excited systems treated as heated ones. The method of temperature (Matsubara) Green's functions [1] is a standard method for studying finite quantum Fermi systems at finite temperature. Equations of the random-phase approximation (RPA) at finite temperature in a nucleus were obtained with the aid of statistical Green's functions [2]. The technique of Matsubara Green's functions was used in generalizing the theory of finite Fermi systems to the case of $T \neq 0$ [3]. There are, however, different statistical approaches, among which so-called thermo-field dynamics (TFD) is worthy of special note [4]. The ultimate TFD version [5] has a number of advantages in relation to the formalism of Matsubara Green's functions. For example, the TFD approach employs, in addition to Green's function and Feynman diagram techniques, operator expansions and the concept of the temperature-dependent vacuum. Within this approach, temperature effects arise consistently through temperature-dependent vertices, and this is convenient for constructing various approximations. Owing to this, approximations of many-body theory that were successful at zero temperature admit quite a straightforward generalization within the TFD approach to the case of finite temperature.

Many approximations applied within the TFD approach ignore the influence that fluctuations exert

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on thermodynamic properties and which lead to the appearance of correlation effects. The present study is aimed at exploring the superfluidity properties of heated Fermi systems within an original approach that was called the static fluctuation approximation (SFA) [6–12]. The SFA method, which relies on a single and controllable approximation, makes it possible to take correctly into account the effect of fluctuations of physical observables and to calculate correlations of various order. This method, which was first formulated for calculating thermodynamic properties of spin systems (see, for example, [6–8]), was then extended to the problem of calculations for other multiparticle systems driven by strong interaction [9–12].

The question of whether superconducting pair correlations may exist in nuclei was first discussed in [13, 14]. On the basis of his approach to the superconductivity problem, Bogolyubov formulated in [13] a variational method for studying the superfluidity of nuclear matter, generalizing Fock's method [15]. At the present time, this approach underlies one of the most efficient microscopic methods for studying the structure of nuclei. Assuming the existence of an energy gap between the ground state and the first excited states of noncollective nature in the spectra of nuclei, A. Bohr, Mottelson, and Pines [14] drew an analogy with the electron spectrum of superconductor metals. In accordance with the superconductivity theory [16, 17], they put forth the assumption that a two-body attractive interaction may also exist between intranuclear nucleons. Pair-correlation effects in finite Fermi systems were considered in [18].

An interaction that may generate superconducting pair correlations of fermions has the form

$$H_{\text{pair}} = -\sum_{q,q'} G(q+,q-;q'-,q'+) \times a^+_{q+}a^+_{q-}a_{q'-}a_{q'+}. \tag{1}$$

As the result of this interaction, two particles in the $(q'+,q'-)$ states that are conjugate with respect to the time-reflection operation go over to the $(q+,q-)$ state. For example, a transition of two particles from one mean-field level to another occurs in deformed nuclei under the effect of the interaction in (1).

Under the assumption that forces that lead to pair fermion correlations are short-range (as is the case for intranuclear nucleons), we can roughly represent them as a delta function of the form $\delta(\mathbf{r}-\mathbf{r}')$. This means that, in momentum space, they are constant. Therefore, the matrix elements of these forces between mean-field eigenfunctions can approximately be taken to be constants as well. This gives sufficient grounds to assume that $G(q+,q-;q'-,q'+)$ does not depend on q and q'; that is,

$$G(q+,q-;q'-,q'+) = G. \tag{2}$$

2. FORMULATION OF THE METHOD AND BASIC EQUATIONS

Let us consider a system of interacting fermions that is described by the Hamiltonian

$$H = \sum_s (E_s-\lambda)a^+_{s\sigma}a_{s\sigma} - G\sum_{s,s'} a^+_{s+}a^+_{s-}a_{s'-}a_{s'+}. \tag{3}$$

The fermion creation and annihilation operators $a^+_{s\sigma}$ and $a_{s'\sigma'}$ satisfy the commutation relations

$$\{a^+_{s\sigma}, a_{s'\sigma'}\} = a^+_{s\sigma}a_{s'\sigma'} + a_{s'\sigma'}a^+_{s\sigma} = \delta_{s,s'}\delta_{\sigma,\sigma'}, \tag{4a}$$

$$\{a_{s\sigma}, a_{s'\sigma'}\} = a_{s\sigma}a_{s'\sigma'} + a_{s'\sigma'}a_{s\sigma} = 0, \tag{4b}$$

$$\{a^+_{s\sigma}, a^+_{s'\sigma'}\} = a^+_{s\sigma}a^+_{s'\sigma'} + a^+_{s'\sigma'}a^+_{s\sigma} = 0, \tag{4c}$$

where E_s stands for single-particle mean-field energies and λ is a chemical potential that is determined from the condition that, on average, the number of fermions, N_f, is conserved; that is,

$$\sum_{s,\sigma}\langle a^+_{s\sigma}a_{s\sigma}\rangle = N_f. \tag{5}$$

We now go over to the Heisenberg representation for the operators $a_{p\sigma}$ and $a^+_{p\sigma}$ by means of the transformation $a^\pm_{p\sigma}(\tau) = e^{\tau H}a^\pm_{p\sigma}e^{-\tau H}$ $(\tau = it)$ and represent the respective equation of motion in the form

$$\frac{da^+_{p\sigma}(\tau)}{d\tau} = [H, a^+_{p\sigma}(\tau)] = \xi_p a^+_{p\sigma}(\tau) + \sigma D^+(\tau)a_{p,-\sigma}(\tau). \tag{6}$$

Here,

$$D^+ = -G\sum_s d^+_s, \quad \xi_p = E_p - \lambda, \tag{7}$$

where $d^+_s = a^+_{s+}a^+_{s-}$ is the creation operator for a pair of fermions in conjugate states. Taking a Hermitian conjugate in Eq. (7) and making the substitution $\sigma \to -\sigma$, we obtain the equation of motion for $a_{p,-\sigma}(\tau)$ in the form

$$\frac{da_{p,-\sigma}(\tau)}{d\tau} = \sigma a^+_{p\sigma}(\tau)D(\tau) - \xi_p a_{p,-\sigma}(\tau), \tag{8}$$

where D is the operator conjugate to the operator D^+. Upon replacing the operators $D^\pm$ in Eqs. (6) and (8) by their expectation values, we obtain the well-known Hartree–Fock–Bogolyubov (HFB) approximation [17], which leads to the independent-quasiparticle model. Below, we will show how one can go beyond this approximation and take into account correlations between quasiparticles.

The application of the SFA procedure to the model being considered is based on the following two assumptions. First, we assume that the operators $D^{\pm}(\tau)$ are integrals of motion; that is, they satisfy the condition

$$\left[D^{\pm}(\tau), H\right] = 0. \tag{9}$$

Second, we assume that the following approximate equality holds:

$$D^{+}D \simeq \left\langle D^{+}D\right\rangle. \tag{10}$$

The approximation in (10) forms the basis of the SFA approach and permits taking into account quasiparticle correlations, which are of importance in systems formed by a finite number of fermions. The equalities in (9) and (10) make it possible to integrate the equation of motion in (6), whereby we obtain the time dependence of the fermion-creation operator $a^{+}_{p\sigma}(\tau)$ in an analytic form; that is,

$$a^{+}_{p\sigma}(\tau) = \Phi_{+}(\tau)a^{+}_{p\sigma} + \sigma\Phi(\tau)D^{+}a_{p,-\sigma}, \tag{11}$$

$$\Phi_{\pm}(\tau) = \cosh\left(\Omega_p\tau\right) \pm \frac{\xi_p \sinh\left(\Omega_p\tau\right)}{\Omega_p}, \tag{12}$$

$$\Phi(\tau) = \frac{\sinh\left(\Omega_p\tau\right)}{\Omega_p},$$

$$\Omega_p = \sqrt{\xi_p^2 + \langle D^{+}D\rangle}. \tag{13}$$

The resulting time dependence in Eq. (11) enables one to derive a set of equations for determining the expectation values of the operators characterizing the system. For this purpose, we make use of the identities

$$\langle A(\beta)B(0)\rangle = \langle B(0)A(0)\rangle, \tag{14a}$$

$$\langle B(0)A(-\beta)\rangle = \langle A(0)B(0)\rangle, \tag{14b}$$

where A and B are arbitrary operators and $\langle A\rangle = \mathrm{tr}\left(\exp\left(-\beta H\right)A\right)\mathrm{tr}^{-1}\left(\exp\left(-\beta H\right)\right)$ means averaging over the equilibrium statistical ensemble, β being the inverse temperature ($\beta = 1/T$). Upon setting, in the equalities in (14), $A(\beta) = a^{+}_{p\sigma}(\beta)$, $B(0) = a_{p\sigma}A$, where one can determine $a^{+}_{p\sigma}(\beta)$ from Eq. (11) at $\tau = \beta$, we arrive at the equations

$$\left\langle a_{p\sigma}a^{+}_{p\sigma}A\right\rangle + \left\langle a_{p\sigma}\left[A, a^{+}_{p\sigma}\right]\right\rangle \tag{15}$$
$$= \Phi_{+}(\beta)\left\langle a^{+}_{p\sigma}a_{p\sigma}A\right\rangle + \sigma\Phi(\beta)\left\langle D^{+}a_{p,-\sigma}a_{p\sigma}A\right\rangle,$$

$$\left\langle a^{+}_{p\sigma}a_{p\sigma}A\right\rangle = \Phi_{-}(\beta)\Big(\left\langle a_{p\sigma}a^{+}_{p\sigma}A\right\rangle \tag{16}$$
$$+ \left\langle a_{p\sigma}\left[A, a^{+}_{p\sigma}\right]\right\rangle\Big) - \sigma\Phi(\beta)\left\langle a_{p\sigma}AD^{+}a_{p,-\sigma}\right\rangle.$$

After the subtraction of Eq. (16) from Eq. (15) and some simple algebra, we obtain the first basic equation in the form

$$\langle n_{p\sigma}A\rangle = \left(\frac{1}{2} - \xi_p\eta_p\right) \tag{17}$$
$$\times \left\{\langle A\rangle + \left\langle a_{p\sigma}\left[A, a^{+}_{p\sigma}\right]\right\rangle\right\}$$
$$- \sigma\eta_p\left\{\left\langle a_{p\sigma}\left[A, D^{+}a_{p,-\sigma}\right]\right\rangle - \sigma G\left\langle n_{p,-\sigma}A\right\rangle\right\},$$

$$\eta_p = \frac{\tanh\left(\beta\Omega_p/2\right)}{2\Omega_p}. \tag{18}$$

We will now find the second basic equation. Setting $A(\beta) = a^{+}_{p\sigma}(\beta)$ and $B(0) = a^{+}_{p,-\sigma}B$ in the identities in (14), we obtain

$$\left\langle a^{+}_{p,-\sigma}a^{+}_{p\sigma}B\right\rangle + \left\langle a^{+}_{p,-\sigma}\left[B, a^{+}_{p\sigma}\right]\right\rangle \tag{19}$$
$$= \Phi_{+}(\beta)\left\langle a^{+}_{p\sigma}a^{+}_{p,-\sigma}B\right\rangle + \sigma\Phi(\beta)\left\langle D^{+}a_{p,-\sigma}a^{+}_{p,-\sigma}B\right\rangle,$$

$$\left\langle a^{+}_{p\sigma}a^{+}_{p,-\sigma}B\right\rangle = \Phi_{-}(\beta)\Big(\left\langle a^{+}_{p,-\sigma}a^{+}_{p\sigma}B\right\rangle \tag{20}$$
$$+ \left\langle a^{+}_{p,-\sigma}\left[B, a^{+}_{p\sigma}\right]\right\rangle\Big) - \sigma\Phi(\beta)\left\langle a^{+}_{p,-\sigma}BD^{+}a_{p,-\sigma}\right\rangle.$$

Subtracting Eq. (20) from Eq. (19) and performing some simple transformations, we obtain the second basic equation in the form

$$\left\langle d^{+}_{p}B\right\rangle = \left(\frac{1}{2} - \xi_p\eta_p\right)\left\langle a^{+}_{p-}\left[B, a^{+}_{p+}\right]\right\rangle \tag{21}$$
$$- \eta_p\left\{\left\langle D^{+}B\right\rangle + \left\langle a^{+}_{p-}\left[B, D^{+}a_{p-}\right]\right\rangle\right\}.$$

We referred to Eqs. (17) and (21) as basic equations since, upon appropriately choosing form for the operators A and B, these equations make it possible to obtain a complete set of equations for all of the required correlation functions for the system being considered. Indeed, expressions for the expectation values of the operators $n_{p\sigma}$ and d^{+}_{p} can be derived upon setting $A = B = 1$ in Eqs. (17) and (21):

$$\langle n_{p\pm}\rangle = \frac{1/2 - \xi_p\eta_p}{1 - G\eta_p}, \tag{22a}$$

$$\left\langle d^{+}_{p}\right\rangle = -\eta_p\left\langle D^{+}\right\rangle. \tag{22b}$$

Equation (22a) makes it possible to obtain an equation for determining the chemical potential:

$$\sum_{p,\sigma}\langle n_{p\sigma}\rangle = N_f \Rightarrow \sum_{p}\frac{1 - 2\xi_p\eta_p}{1 - G\eta_p} = N_f. \tag{23}$$

With the aid of Eq. (22b), we can obtain a basic equation of Bardeen–Cooper–Schrieffer (BCS) theory [16]; that is,

$$\left\langle D^{+}\right\rangle = \left\langle D^{+}\right\rangle G\sum_{p}\eta_p \tag{24}$$

or

$$\frac{G}{2}\sum_p \frac{\tanh\left(\beta\sqrt{\xi_p^2+\langle D^+D\rangle}/2\right)}{\sqrt{\xi_p^2+\langle D^+D\rangle}} = 1.$$

Allowance for correlations between quasiparticles (pair of fermions in conjugate states) makes it possible to go beyond BCS or HFB theory [16, 17] and to derive an energy-gap equation that takes into account gap fluctuations.

Setting $A = n_{q\sigma'}$ and $B = d_q$ in Eqs. (17) and (21), we arrive at a closed set of equations for determining the pair correlation functions $\langle n_{p\sigma}n_{q\sigma'}\rangle$ and $\langle d_p^+ d_q\rangle$. Specifically, we have

$$\begin{aligned}\langle n_{p\sigma}n_{q\sigma'}\rangle &= G\eta_p\langle n_{p,-\sigma}n_{q\sigma'}\rangle \\ &+ \langle n_{p\sigma}\rangle(1-G\eta_p)\left\{\langle n_{q\sigma'}\rangle + \delta_{pq}\delta_{\sigma\sigma'}(1-\langle n_{p\sigma}\rangle)\right\} \\ &- \eta_p\left\{G\langle d_q^+ d_p\rangle + \delta_{pq}\delta_{\sigma,-\sigma'}\langle D^+ d_p\rangle\right\},\end{aligned} \quad (25)$$

$$\begin{aligned}\langle d_p^+ d_q\rangle &= \delta_{pq}(1-G\eta_p)\langle n_p\rangle^2 - \eta_p\langle D^+ d_q\rangle \\ &- G\eta_p\left\{\langle n_{p-}(n_{q+}+n_{q-})\rangle - (1+\delta_{pq})\langle n_p\rangle\right\}.\end{aligned} \quad (26)$$

Knowing the expression for the correlation function $\langle d_p^+ d_q\rangle$, one can obtain an equations for the energy gap,

$$\Delta^2 = \langle D^+D\rangle = G^2\sum_{p,q}\langle d_p^+ d_q\rangle, \quad (27)$$

and the mean energy,

$$\langle H\rangle = 2\sum_p \xi_p\langle n_p\rangle - \frac{\Delta^2}{G}. \quad (28)$$

3. ENERGY GAP

The present section is devoted to deriving an equation for the energy gap, which is determined by the correlation function $\langle D^+D\rangle$ according to (27). For this purpose, we multiply both sides of Eq. (26) by G^2 and perform summation over momenta. As result, we arrive at the equation

$$\begin{aligned}\langle D^+D\rangle &= G\langle D^+D\rangle\sum_p \eta_p \\ &+ G^2\sum_p(1-G\eta_p)\langle n_p\rangle^2 \\ &+ G^3\left\{(N/2+1)\sum_p\eta_p\langle n_p\rangle - \sum_p\eta_p\left\langle n_{p-}\hat{N}_f\right\rangle\right\},\end{aligned} \quad (29)$$

where $\hat{N}_f = \sum_q(n_{q+}+n_{q-})$ is the fermion-number operator and N is the maximum number of particles. The correlation function $\left\langle n_{p-}\hat{N}_f\right\rangle$ appearing in Eq. (29) can be determined from Eq. (25) or from Eq. (17) upon setting $A = \hat{N}_f$. Choosing the second way and performing some simple transformations, we obtain

$$\begin{aligned}\left\langle n_{p-}\hat{N}_f\right\rangle &\\ &= \langle n_p\rangle\left\{N_f + \frac{1}{1-G\eta_p} - \langle n_p\rangle\right\} \\ &+ \frac{\eta_p}{1-G\eta_p}\langle D^+ d_p\rangle.\end{aligned} \quad (30)$$

Substituting expression (30) in Eq. (29), we ultimately obtain an energy-gap equation in the form

$$\begin{aligned}\langle D^+D\rangle &\\ &= \sum_p G\eta_p\left\{\langle D^+D\rangle - \frac{G^2\eta_p}{1-G\eta_p}\langle D^+ d_p\rangle\right\} \\ &+ G^2\sum_p\langle n_p\rangle\left\{\langle n_p\rangle + G\eta_p\left(\frac{N}{2}\right.\right. \\ &\left.\left. + 1 - N_f - \frac{1}{1-G\eta_p}\right)\right\}.\end{aligned} \quad (31)$$

It is rather difficult to analyze Eq. (31); therefore, we simplify it, considering that $G\eta_p \ll 1$. Equation (31) then assumes the form

$$\begin{aligned}\Delta^2 &= \Delta^2 G\sum_p\eta_p \\ &+ G^2\sum_p\langle n_p\rangle\left\{\langle n_p\rangle + G\eta_p\left(\frac{N}{2}-N_f\right)\right\},\end{aligned} \quad (32)$$

$$\langle n_p\rangle \approx \frac{1}{2} - \xi_p\eta_p. \quad (33)$$

Equations (5), (32), and (33) form a closed set of equations for determining the correlation function $\langle D^+D\rangle$ and the chemical potential λ.

Let us consider the particular case where, in the energy range between E_1 and E_2, the density of single-particle levels in the mean field is constant and is equal to $\bar{\rho}$. The total number of levels in this range is $N/2 = \bar{\rho}(E_2 - E_1)$. In Eqs. (5) and (32), we go over from summation to integration and represent them in the form

$$\bar{\rho}\int_{E_1}^{E_2}(1-(E-\lambda)F(\Delta, E-\lambda, T))\,dE = N_f, \quad (34)$$

$$\Delta^2 = \Delta^2 \frac{G\bar{\rho}}{2} \int_{E_1}^{E_2} F(\Delta, E-\lambda, T) dE \tag{35}$$
$$+ \frac{G^2}{4} \left\{ N_f + \bar{\rho} \int_{E_1}^{E_2} F(\Delta, E-\lambda, T)\big(1 - (E-\lambda) \times F(\Delta, E-\lambda, T)\big)\left(G(N/2 - N_f) - (E-\lambda)\right) dE \right\},$$

where we have introduced the notation

$$F(\Delta, E-\lambda, T) = \frac{\tanh\left[\left(\sqrt{\Delta^2 + (E-\lambda)^2}\right)/2T\right]}{\sqrt{\Delta^2 + (E-\lambda)^2}}. \tag{36}$$

It is hardly possible to construct an explicit analytic solution of the integral equations (35) and (36); therefore, we will solve them numerically, preliminarily analyzing at $T = 0$ the equations that we obtained. In the case of $T = 0$, Eqs. (35) and (36) assume the form

$$\bar{\rho} \int_{E_1}^{E_2} \left(1 - \frac{(E-\lambda)}{\sqrt{\Delta^2 + (E-\lambda)^2}}\right) dE = N_f, \tag{37}$$

$$\Delta^2 = \Delta^2 \frac{G\bar{\rho}}{2} \int_{E_1}^{E_2} \frac{dE}{\sqrt{\Delta^2 + (E-\lambda)^2}} \tag{38}$$
$$+ \frac{G^2}{4} \left\{ N_f + \bar{\rho} \int_{E_1}^{E_2} \left(1 - \frac{(E-\lambda)}{\sqrt{\Delta^2 + (E-\lambda)^2}}\right) \times \frac{(G(N/2 - N_f) - (E-\lambda))\, dE}{\sqrt{\Delta^2 + (E-\lambda)^2}} \right\}.$$

After integration in Eqs. (37) and (38) and some algebra, we arrive at

$$\Delta = \frac{\sqrt{N_f(N - N_f)}}{2\bar{\rho}\sinh x}, \tag{39}$$

where x satisfies the transcendental equation

$$x = \frac{1}{\theta} - \frac{2\theta N^{-1}\cosh a \cdot \sinh x \left[2\cosh a \cdot \sinh x + \arctan e^{a+x} - \arctan e^{a-x} + \theta \sinh a \cdot \sinh x \ln \frac{\cosh(a+x)}{\cosh(a-x)}\right]}{1 + 2N^{-1}\theta^2 \sinh 2a \cdot \sinh^2 x}, \tag{40}$$

in which

$$a = \frac{1}{2}\ln\frac{1-n}{n}, \quad n = \frac{N_f}{N}, \tag{41}$$

and $\theta = G\bar{\rho}$. From Eq. (39), one can see that, at $N_f = 0$ and $N_f = N$, there are no correlations in just the same way as in the HFB approximation. It is noteworthy that, in BCS theory, we have $x_{\text{BCS}} = \theta^{-1}$.

We also present an expression for the ground-state energy. We have

$$E_0 = \frac{N_f^2}{4\bar{\rho}} \left\{\coth x + \frac{1-n}{n\sinh^2 x}\left(x - \frac{1}{\theta}\right)\right\}. \tag{42}$$

One can derive it by employing expression (28) for the mean energy and Eqs. (39) and (33) upon setting $T = 0$ in them. We note that, in BCS theory, the ground-state energy is

$$E_0^{\text{BCS}} = \left(N_f^2/4\bar{\rho}\right)\coth\left(\theta^{-1}\right).$$

By way of illustration, the dependences of the quantity x, the energy gap Δ, and the ground-state energy E_0 on the parameter θ are given in Figs. 1, 2, and 3, respectively, at the number of single-particle levels that is equal to $N/2 = 50$ for various values of the band-filling number n. The graphs on display show that the results based on the application of the SFA method deviate substantially from their counterparts obtained by the HFB method at small values of the parameter θ, which correspond to deformed nuclei, the SFA method leading to a higher value of the energy gap in relation to the HFB method (see Fig. 2). However, only for $n < 1/2$ is the reduction of the ground-state energy observed, as can be seen from Fig. 3.

We now proceed to analyze the model under study for $T \neq 0$. The normalized energy gap $\Delta(T)/\Delta(0)$ as a function of the reduced temperature T/T_c^{BCS}, where T_c^{BCS} is the critical temperature in BCS theory, is shown in Fig. 4 at the number of single-particle levels that is equal to $N/2 = 50$ and at the parameter value of $\theta = 0.5$ for various values of the band-filling num-

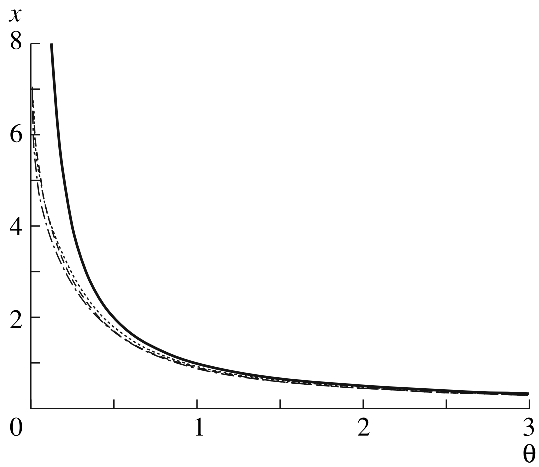

Fig. 1. x as a function of the parameter θ at $N = 100$ for various values of the band-filling number n: (dashed curve) $n = 0.2$, (dotted curve) $n = 0.5$, and (dash-dotted curve) $n = 0.8$. The thick solid curve corresponds to BCS theory ($x_{\rm BCS} = \theta^{-1}$).

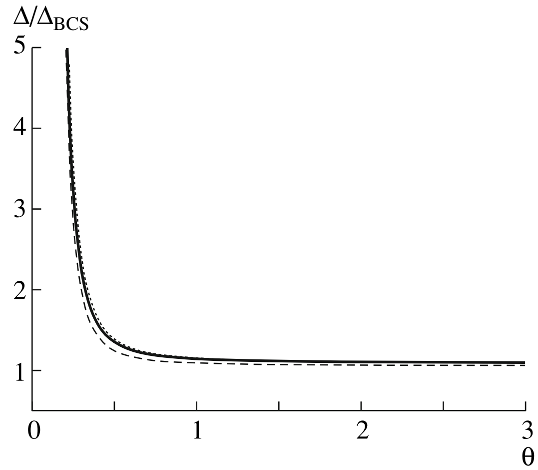

Fig. 2. Normalized energy gap $\Delta/\Delta_{\rm BCS}$ as a function of the parameter θ at $N = 100$ for various values of the band-filling number n: (thick solid curve) $n = 0.2$, (dashed curve) $n = 0.5$, and (dotted curve) $n = 0.8$.

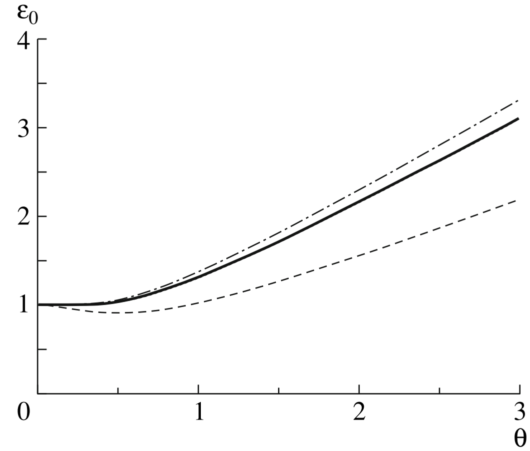

Fig. 3. Normalized ground-state energy $\varepsilon_0 = 4\bar{\rho}E_0/N_f^2$ as a function of the parameter θ at $N = 100$ for various values of the band-filling number n. The notation for the curves is identical to that in Fig. 1.

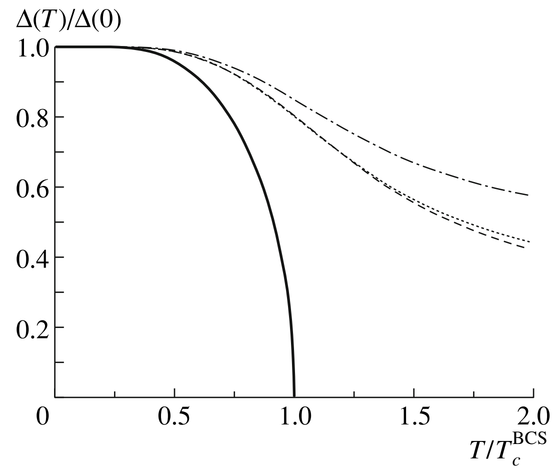

Fig. 4. Normalized energy gap $\Delta(T)/\Delta(0)$ as a function of the relative temperature $T/T_c^{\rm BCS}$ at $N = 100$ and $\theta = 0.5$ for various values of the band-filling number n: (dashed curve) $n = 0.3$, (dotted curve) $n = 0.5$, and (dash-dotted curve) $n = 0.8$. The thick solid curve corresponds to BCS theory.

ber n. For the sake of comparison, the temperature dependence of the energy gap in BCS theory is also presented in this figure. One can see that, within the SFA method, the energy gap does not vanish at any finite temperature. This is a direct result of taking into account gap fluctuations, which lead to the smearing of the phase transition and to the appearance of the so-called fluctuation tail.

4. CONCLUSIONS

In the present study, we have proposed a new method for analyzing superfluidity in a heated finite system of fermions. This method makes it possible to go beyond the independent-quasiparticle model, which is used to study pair correlations of the superconductor type. The method in question relies of the approximation specified in Eq. (10), whereby one can take into account the effect of fluctuations on superfluidity properties. It is the inclusion of fluctuations that leads to the smearing of the phase transition in the system to a superfluid state.

The proposed method for studying superconductor correlations in finite Fermi systems ia applicable in the case of superconductor interaction potentials that are more realistic than the simplest pairing-interaction form used in the present study.

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