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The Positive Solutions to Quasilinear Elliptic Inequalities on Model Riemannian Manifolds

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Abstract—We investigate the problem of implementation of the Liouville type theorems on the existence of positive solutions of some quasilinear elliptic inequalities on model (spherically symmetric) Riemannian manifolds. In particular, we find exact conditions for the existence and nonexistence of entire positive solutions of the studied inequalities on the Riemannian manifolds. The method is based on a study of radially symmetric solutions of an ordinary differential equation generated by the basic inequality and establishing the relationship of the existence of entire positive solutions of quasilinear elliptic inequalities and solvability of the Cauchy problem for this equation. In addition, in the paper we apply classical methods of the theory of elliptic equations and second order inequalities (the maximum principle, the principle of comparison, etc.). The obtained results generalize similar results obtained previously by Y. Naito and H. Usami for Euclidean space $\mathbf{R}^n$, as well as some earlier results by A. G. Losev and E. A. Mazepa.

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INTRODUCTION

In this paper we investigate the problem of the existence of entire positive solutions of the inequalities of the form

$$Lu \equiv \operatorname{div}(A(|\nabla u|)\nabla u) \geq f(x, u), \tag{1}$$

where the function $f(\cdot, u) \geq 0$ for $u \geq 0$ is continuous in both arguments, $f \not\equiv 0$ and $f(\cdot, 0) = 0$ on noncompact Riemannian manifolds of a special type.

One of the sources of the mentioned problems is the classification theory of noncompact Riemannian surfaces and manifolds; the general information on modern directions of investigation in this area can be found, e.g., in [1]. A specific feature of the surfaces and manifolds of parabolic type is the fulfillment of the Liouville theorem, that states that any positive superharmonic function on the given surface (on the given manifold) is identically constant.

In the recent years, a number of papers was published (see, e.g., [2–8]), that investigate the problems of the existence of entire solutions of various linear and quasilinear equations and inequalities on noncompact Riemannian manifolds (in particular, on model manifolds) and in the Euclidean space, along with their asymptotic behavior. Let us give a detailed description of model manifolds.

Fix the origin $O \in \mathbf{R}^n$ and a smooth function q on the interval $[0, \infty)$ such that $q(0) = 0$ and $q'(0) = 1$. Define a model Riemannian manifold M_q in the following way:

1) the set of points M_q is the entire $\mathbf{R}^n$;

2) in polar coordinates (r, θ) (where $r \in (0, \infty)$ and $\theta \in S^{n-1}$), the Riemannian metric on $M_q \setminus \{O\}$ is defined as

$$ds^2 = dr^2 + q^2(r)d\theta^2, \tag{2}$$

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