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Conditional Gradient Method Without Line-Search

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Abstract—We propose a simple rule for the step-size choice in the conditional gradient method, which does not require any line-search procedure. It takes into account the current behavior of the method. Its convergence is established under the same assumptions as those for the previously known methods.

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INTRODUCTION

The usual optimization problem consists in finding the minimal value of some goal function $f : X \to \mathbb{R}$ on a feasible set D such that $D \subseteq X \subseteq \mathbb{R}^n$. For brevity, we write this problem as

$$\min_{x \in D} \to f(x), \tag{1}$$

its solution set is denoted by D^* and the optimal value of the function by f^*, i.e., $f^* = \inf_{x \in D} f(x)$. We will consider the well-known class of optimization problems, where the set D is supposed to be nonempty, convex, and compact and the function f is supposed to be convex and smooth on the set D. Then the set D^* is also nonempty, convex, and compact. This problem can be solved with many methods, including the well-known conditional gradient method, which was suggested in [1] and further was developed in [2, 3]. For a long period it was not considered as very efficient due to its relatively slow convergence. However, it has gained now a great amount of attention due to several useful features such as its simpler auxiliary problem in comparison with the projection onto D and sparsity of its current iteration points, which appeared especially significant in solution of huge dimensionality problems with inexact data. Many efforts were concentrated on enhancing its convergence properties (e.g., [4, 5]).

One of the main questions in implementation of the conditional gradient method as well as any other iterative method is the step-size choice rule. In fact, exact or approximate one-dimensional minimization requires great computational expenses per iteration especially in the case where calculation of the function value is almost similar to calculation of its derivative (gradient) and needs solving of complex auxiliary problems. Application of step-size rules, which do not utilize the information about the current behavior of the problem, for instance,

$$\sum_{k=0}^{\infty} \lambda_k = \infty,\ \sum_{k=0}^{\infty} \lambda_k^2 < \infty,\ \lambda_k \in (0,1),\ k = 1, 2, \dots,$$

(see, e.g., [6]) usually yields slow convergence. When utilizing a priori information such as Lipschitz constants, one must take into account that only their inexact estimates are known, which also leads to slow convergence.

In this paper we propose a simple step-size choice rule for the conditional gradient method, which does not require any line-search but utilizes the information about the current behavior of the method. Its convergence is established under the same assumptions as those in the known versions of the method.

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