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IMPROVED NONPARAMETRIC ESTIMATION OF THE DRIFT IN DIFFUSION PROCESSES

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Abstract

In this paper, we have considered the robust adaptive nonparametric estimation problem for the drift coefficient in diffusion processes. It has been shown that the initial estimation problem can be reduced to the estimation problem in a discrete time nonparametric heteroscedastic regression model by using the sequential approach. We have developed a new sharp model selection method for estimating the unknown drift function using the improved estimation approach. An adaptive model selection procedure based on the improved weighted least square estimates has been proposed. It has been established that such estimate outperforms in non-asymptotic mean square accuracy the procedure based on the classical weighted least square estimates. Sharp oracle inequalities for the robust risk have been obtained.

Keywords: improved estimation, stochastic diffusion process, mean-square accuracy, model selection, sharp oracle inequality

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Introduction

Let $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, \mathbf{P})$ be a filtered probability space on which the following stochastic differential equation is defined:

$$\mathrm{d}y_t = S(y_t)\,\mathrm{d}t + \mathrm{d}w_t, \quad 0 \leq t \leq T, \tag{1}$$

where $(w_t)_{t\geq 0}$ is a scalar standard Wiener process, the initial value y_0 is some given constant, and $S(\cdot)$ is an unknown function.The problem is to estimate the function $S(x)$, $x \in [a, b]$, from the observations $(y_t)_{0\leq t\leq T}$. The calibration problem for the model (1) is important in various applications. In particular, it appears, when constructing optimal strategies for the investor behavior in diffusion financial markets. It is known that the optimal strategy depends on unknown market parameters, in particular, on unknown drift coefficient S. Therefore, in practical financial calculations it is necessary to use statistical estimates for the function S which are reliable on some fixed time interval $[0, T]$ [1]. Earlier, the problem of non-asymptotic estimation of the parameters of diffusion processes was studied in [2]. Here, it was shown that many difficulties of asymptotic estimation of parameters for one-dimensional diffusion processes can be overcome by using a sequantial approach. It turns out that the theoretical analysis of successive estimates is simpler than the analysis of classical procedures. In particular, it is possible to calculate non-asymptotic bounds for quadratic risk. Owing to the use of a sequential approach, the problems of non-asymptotic estimation of parameters were studied in [3] for multidimensional diffusion processes and, recently, in [4] for multidimensional continuous and discrete semimartingales. In [5], a truncated sequential method for estimating the parameters of diffusion processes was developed. Nonparametric estimation has been covered in a number of publications. A consistent approach to nonparametric criteria for minimax estimation of the drift coefficient in (ergodic) diffusion processes was developed in [6]. In this paper, sequential pointwise kernel estimates are considered. For such estimates, non-asymptotic upper bounds of the root-mean-square risk are obtained, and these estimates give the optimal convergence rate as $T \to \infty$.

The present paper deals with estimation of the unknown function $S(x)$, $a \le x \le b$, in the sense of the mean square risk

$$\mathcal{R}(\widehat{S}_T, S) = \mathbf{E}_S \|\widehat{S}_T - S\|^2, \quad \|S\|^2 = \int_a^b S^2(x)\mathrm{d}x, \tag{2}$$

where $\widehat{S}_T$ is the estimate of S by observations $(y_t)_{0\le t\le T}$, $a < b$ are some real numbers. Here $\mathbf{E}_S$ is the expectation with respect to the distribution $\mathbf{P}_S$ of the random process $(y_t)_{0\le t\le T}$ given the drift function S.

The purpose of this paper is to construct an adaptive estimate S^* of the drift coefficient S in (1) and to show that the quadratic risk of this estimate is less than the one of the estimate proposed in [6], i.e., we construct the improved estimate in the mean square accuracy sense. In order to fulfill this purpose, we use the improved estimation approach proposed in [7] and [8] for parametric regression models and recently developed in [9] for a nonparametric estimation problem. Moreover, we consider the estimation problem in adaptive setting, i.e., when the regularity of S is unknown, by using a model selection method proposed in [10]. This approach provides an adaptive solution for the nonparametric estimation through oracle inequalities, which give the nonparametric upper bound for the quadratic risk of estimate.

1. Passage to a discrete time regression model

To obtain a reliable estimate of the function S, it is necessary to impose on it certain conditions that are analogous to the periodicity of the deterministic signal in the white noise model [11]. One of the conditions sufficient for this purpose is the assumption that the process $(y_t)_{t\ge 0}$ in (1) returns to any neighborhood of each points $x \in [a, b]$. As in [6], in order to get the ergodicity of the process (1), we define the following functional class:

$$\Sigma_{L,N} = \{S \in \mathrm{Lip}_L(\mathbb{R}) : |S(N)| \le L; \quad \forall\, |x| \ge N,\ \exists\, \dot{S}(x) \in \mathbf{C}(\mathbb{R})$$
$$\text{such that} - L \le \inf_{|x|\ge N} \dot{S}(x) \le \sup_{|x|\ge N} \dot{S}(x) \le -1/L\}, \tag{3}$$

where $L > 1$, $N > |a| + |b|$, $\dot{S}(x)$ is the derivative $S(x)$,

$$\mathrm{Lip}_L(\mathbb{R}) = \left\{ f \in \mathbf{C}(\mathbb{R}) : \sup_{x,y\in\mathbb{R}} \frac{|f(x) - f(y)|}{|x - y|} \le L \right\}.$$

We note that if $S \in \Sigma_{L,N}$, then there exists an invariant density

$$q(x) = q_S(x) = \exp\left\{ 2\int_0^x S(z)\,\mathrm{d}z \right\} \Big/ \int_{-\infty}^{+\infty} \exp\left\{ 2\int_0^y S(z)\,\mathrm{d}z \right\} \mathrm{d}y. \tag{4}$$

We note that the functions in $\Sigma_{L,N}$ are uniformly bounded on $[a, b]$, i.e.

$$s^* = \sup_{a\le x\le b}\ \sup_{S\in\Sigma_{L,N}} S^2(x) < \infty.$$

We start with the partition of the interval $[a,b]$ by the points $(x_k)_{1\le k\le n}$, defined as

$$x_k = a + \frac{k}{n}(b-a), \tag{5}$$

where $n = n(T)$ is an integer-valued function of T, such that

$$n(T) \le T \quad \text{and} \quad \lim_{T\to\infty} \frac{n(T)}{T} = 1. \tag{6}$$

Now, at any point x_k, we estimate the function S by sequential kernel estimation. We fix some $0 < t_0 < T$ and put

$$\begin{cases} \tau_k = \inf\left\{ t \ge t_0 : \int\limits_{t_0}^{t} Q\left(\frac{y_s - x_k}{h}\right) \mathrm{d}s \ge H_k \right\}; \\ \widetilde{S}_k = \frac{1}{H_k} \int\limits_{t_0}^{\tau_k} Q\left(\frac{y_s - x_k}{h}\right) \mathrm{d}y_s, \end{cases} \tag{7}$$

where $Q(z) = \mathbf{1}_{\{|z|\le 1\}}$, $\mathbf{1}_A$ is an indicator of the set A, $h = (b-a)/(2n)$ and H_k is a positive threshold, which will be indicated below. From (1), it is easy to obtain that

$$\widetilde{S}_k = S(x_k) + \zeta_k.$$

The error ζ_k is represented as a sum of the approximating and stochastic parts, i.e.,

$$\zeta_k = B_k + \frac{1}{\sqrt{H_k}}\xi_k, \quad B_k = \frac{1}{H_k}\int\limits_{t_0}^{\tau_k} Q\left(\frac{y_s - x_k}{h}\right) \Delta S(y_s, x_k)\,\mathrm{d}s,$$

where $\Delta S(y,x) = S(y) - S(x)$ and

$$\xi_k = \frac{1}{\sqrt{H_k}} \int\limits_{t_0}^{\tau_k} Q\left(\frac{y_s - x_k}{h}\right) \mathrm{d}w_s.$$

Taking into account that S is the Lipshitz function, we obtain an upper bound for the approximating part as

$$|B_k| \le Lh.$$

It is easy to see that random variables $(\xi_k)_{1\le k\le n}$ are independent identically distributed from $\mathcal{N}(0,1)$. In [6], it is established that an effective kernel estimate of the form (7) has a stochastic part distributed as $\mathcal{N}(0, 2Thq_S(x_k))$, where $q_S(x_k)$ is the ergodic density defined in (4). Therefore, for an effective estimate at each point x_k by the kernel estimate (7), we need to estimate the density (4) from observations $(y_t)_{0\le t\le t_0}$ .To this end, we establish that

$$\widetilde{q}_T(x_k) = \max\{\widehat{q}(x_k), \epsilon_T\},$$

where ϵ_T is positive, $0 < \epsilon_T < 1$,

$$\widehat{q}(x_k) = \frac{1}{2t_0 h} \int\limits_0^{t_0} Q\left(\frac{y_s - x_k}{h}\right) \mathrm{d}s.$$

Now, we choose the threshold H_k in (7):

$$H_k = (T - t_0)\left(2\widetilde{q}_T(x_k) - \epsilon_T^2\right) h.$$

Let us suppose that the parameters $t_0 = t_0(T)$ and ϵ_T satisfy the following conditions:

$\mathbf{H}_1$) For any $T \ge 32$,

$$16 \le t_0 \le T/2 \quad \text{and} \quad \sqrt{2}/t_0^{1/8} \le \epsilon_T \le 1.$$

$\mathbf{H}_2$)

$$\lim_{T\to\infty} t_0(T) = \infty, \quad \lim_{T\to\infty} \epsilon_T = 0, \quad \lim_{T\to\infty} T\epsilon_T/t_0(T) = \infty.$$

$\mathbf{H}_3$) For any $\nu > 0$ and $m > 0$,

$$\lim_{T\to\infty} T\epsilon_T^m = \infty \quad \text{and} \quad \lim_{T\to\infty} T^m e^{-\nu\sqrt{t_0}} = 0.$$

For example, for $T \ge 32$,

$$t_0 = \max\{\min\{\ln^4 T, T/2\}, 16\} \quad \text{and} \quad \epsilon_T = \sqrt{2}\, t_0^{-1/8}.$$

Let

$$\Gamma = \{\max_{1\le l\le n} \tau_l \le T\} \quad \text{and} \quad Y_k = \widetilde{S}_k \mathbf{1}_\Gamma. \tag{8}$$

Then, there exists a temporary heteroscedastic regression model on the set Γ

$$Y_k = S(x_k) + \zeta_k, \quad \zeta_k = \sigma_k \xi_k + \delta_k \tag{9}$$

with $\delta_k = B_k$ and

$$\sigma_k^2 = \frac{n}{(T - t_0)(\widetilde{q}_T(x_k) - \epsilon_T^2/2)(b-a)}.$$

It should be noted that from (6) and $\mathbf{H}_1$), we get the following upper bound

$$\max_{1\le k\le n} \sigma_k^2 \le \frac{4}{(b-a)\epsilon_T} = \sigma_* \tag{10}$$

for which, by condition $\mathbf{H}_3$),

$$\lim_{T\to\infty} \frac{\sigma_*}{T^m} = 0 \quad \text{for any} \quad m > 0.$$

To estimate the function S from the observations of (9), we should study some properties of the set Γ in (8).

Proposition 1. *Let us suppose that the parameters t_0 and ϵ_T satisfy the following conditions: $\mathbf{H}_1$)–$\mathbf{H}_3$). Then*

$$\sup_{S\in\Sigma_{L,N}} \mathbf{P}_S(\Gamma^c) \le \Pi_T,$$

where $\lim_{T\to\infty} T^m \Pi_T = 0$ *for any* $m > 0$.

2. Improved estimates

In this section, we consider the estimation problem for the model (9). The function $S(\cdot)$ is unknown and has to be estimated from observations $Y_1,\ldots,Y_n$.

The accuracy of any estimator $\widehat{S}$ will be measured by the empirical squared error of the form

$$\|\widehat{S}-S\|_n^2=(\widehat{S}-S,\widehat{S}-S)_n=\frac{b-a}{n}\sum_{l=1}^{n}(\widehat{S}(x_l)-S(x_l))^2.$$

Now, we fix a basis $(\phi_j)_{1\le j\le n}$, which is orthonormal for the empirical inner product:

$$(\phi_i,\,\phi_j)_n=\frac{b-a}{n}\sum_{l=1}^{n}\phi_i(x_l)\phi_j(x_l)=\mathbf{Kr}_{ij},$$

where $\mathbf{Kr}_{ij}$ is Kronecker's symbol. By making use of this basis, we apply the discrete Fourier transformation to (9) and obtain the Fourier coefficients and their least square estimates

$$\theta_{j,n}=\frac{b-a}{n}\sum_{l=1}^{n}S(x_l)\,\phi_j(x_l),\quad \widehat{\theta}_{j,n}=\frac{b-a}{n}\sum_{l=1}^{n}Y_l\,\phi_j(x_l).$$

From (9), it follows directly that these Fourier coefficients satisfy the following equation

$$\widehat{\theta}_{j,n}=\theta_{j,n}+\zeta_{j,n}\quad\text{with}\quad \zeta_{j,n}=\sqrt{\frac{b-a}{n}}\xi_{j,n}+\delta_{j,n},$$

where

$$\xi_{j,n}=\sqrt{\frac{b-a}{n}}\sum_{l=1}^{n}\sigma_l\xi_l\phi_j(x_l)\quad\text{and}\quad \delta_{j,n}=\frac{b-a}{n}\sum_{l=1}^{n}\delta_l\,\phi_j(x_l).$$

Note that the upper bound (10) and the Bounyakovskii–Cauchy–Schwarz inequality imply that

$$|\delta_{j,n}|\le\|\delta\|_n\,\|\phi_j\|_n=\|\delta\|_n.$$

We estimate the function S in (9) on the sieve (5) by the weighted least squares estimator

$$\widehat{S}_\lambda(x_l)=\sum_{j=1}^{n}\lambda(j)\,\widehat{\theta}_{j,n}\,\phi_j(x_l)\,\mathbf{1}_\Gamma,\quad 1\le l\le n,$$

where the weight vector $\lambda=(\lambda(1),\ldots,\lambda(n))$ belongs to some finite set $\Lambda\subset[0,1]^n$. We set for any $a\le x\le b$

$$\widehat{S}_\lambda(x)=\widehat{S}_\lambda(x_1)\mathbf{1}_{\{a\le x\le x_1\}}+\sum_{l=2}^{n}\widehat{S}_\lambda(x_l)\mathbf{1}_{\{x_{l-1}<x\le x_l\}}.\tag{11}$$

Hereafter, we suppose that the first $d\le n$ components of the weight vector λ are equal to 1, i.e., $\lambda(j)=1$ for any $1\le j\le d$.

We consider a new estimate for the function S in (9) of the form

$$S^*_\lambda(x_l)=\sum_{j=1}^{n}\lambda(j)\,\theta^*_{j,n}\,\phi_j(x_l)\,\mathbf{1}_\Gamma,\quad 1\le l\le n,$$

where

$$\theta^*_{j,n} = \left(1 - \frac{c(d)}{\|\widetilde{\theta}_n\|}\mathbf{1}_{\{1\le j\le d\}}\right)\widehat{\theta}_{j,n},$$

where

$$c(d) = \frac{(d-1)\sigma_*^2 L(b-a)^{1/2}}{n(s^* + \sqrt{d\sigma_*/n})}, \quad \|\widetilde{\theta}_n\|^2 = \sum_{j=1}^{d}\widehat{\theta}^2_{j,n}.$$

Now, we define the estimate for S in (1). We set for any $a \le x \le b$

$$S^*_\lambda(x) = S^*_\lambda(x_1)\mathbf{1}_{\{a\le x\le x_1\}} + \sum_{l=2}^{n} S^*_\lambda(x_l)\mathbf{1}_{\{x_{l-1}<x\le x_l\}}. \tag{12}$$

We denote the difference of quadratic risks of the estimates (12) and (11) as

$$\Delta_n(S) := \mathbf{E}_S\|S^*_\lambda - S\|^2_n - \mathbf{E}_S\|\widehat{S}_\lambda - S\|^2_n.$$

The choice of estimate (12) is motivated by the desire to control the quadratic risk.

Theorem 1. *The estimate* (12) *outperforms in the mean square accuracy the estimate* (11), *i.e.,*

$$\sup_{S\in\Sigma_{L,N}} \Delta_n(S) < -c^2(d).$$

3. Oracle inequalities

In order to obtain a good estimator, we have to write a rule to choose a weight vector $\lambda \in \Lambda$ in (12). It is obvious that the best way is to minimize the empirical squared error with respect to λ:

$$\mathrm{Err}_n(\lambda) = \|S^*_\lambda - S\|^2_n \to \min.$$

Making use of (12) and the Fourier transformation of S implies

$$\mathrm{Err}_n(\lambda) = \sum_{j=1}^{n}\lambda^2(j)\theta^{*2}_{j,n} - 2\sum_{j=1}^{n}\lambda(j)\theta^*_{j,n}\theta_{j,n} + \sum_{j=1}^{n}\theta^2_{j,n}.$$

Since the coefficient $\theta_{j,n}$ is unknown, we need to replace the term $\theta^*_{j,n}\theta_{j,n}$ by some estimator, which we choose as

$$\widetilde{\theta}_{j,n} = \widehat{\theta}_{j,n}\theta^*_{j,n} - \frac{b-a}{n}s_{j,n} \quad \text{with} \quad s_{j,n} = \frac{b-a}{n}\sum_{l=1}^{n}\sigma^2_l\phi^2_j(x_l).$$

One has to pay a penalty for this substitution in the empirical squared error. Finally, we define the cost function of the form

$$J_n(\lambda) = \sum_{j=1}^{n}\lambda^2(j)\theta^{*2}_{j,n} - 2\sum_{j=1}^{n}\lambda(j)\widetilde{\theta}_{j,n} + \rho P_n(\lambda),$$

where the penalty term is defined as

$$P_n(\lambda) = \frac{b-a}{n}\sum_{j=1}^{n}\lambda^2(j)s_{j,n}$$

and $0 < \rho < 1$ is some positive constant which will be chosen later. We set

$$\widehat{\lambda} = \underset{\lambda \in \Lambda}{\operatorname{argmin}}\, J_n(\lambda)$$

and define an estimator of S of the form (11):

$$S^*(x) = S^*_{\widehat{\lambda}}(x) \quad \text{for} \quad a \le x \le b. \tag{13}$$

Now, we obtain the non asymptotic upper bound for the quadratical risk of the estimator (13).

Theorem 2. *Let $\Lambda \subset [0,1]^n$ be any finite set such that the first -$d \le n$ components of the weight vector λ are equal to 1. Then, for any $n \ge 3$ and $0 < \rho < 1/6$, the estimator* (13) *satisfies the following oracle inequality*

$$\mathbf{E}_S \|S^* - S\|_n^2 \le \frac{1+6\rho}{1-6\rho} \min_{\lambda \in \Lambda} \mathbf{E}_S \|\widehat{S}_\lambda - S\|_n^2 + \frac{\Psi_n(\rho)}{n},$$

where $\lim_{n\to\infty} \Psi_n(\rho)/n = 0$.

Now, we consider the estimation problem (1) via model (9). We apply the estimating procedure (13) with special weight set introduced in [6] to the regression scheme (9). Denoting $S^*_\alpha = S^*_{\lambda_\alpha}$ we set

$$S^* = S^*_{\widehat{\alpha}} \quad \text{with} \quad \widehat{\alpha} = \underset{\alpha \in \mathcal{A}_\varepsilon}{\operatorname{argmin}}\, J_n(\lambda_\alpha).$$

We obtain through theorem 2 the following oracle inequality.

Theorem 3. *Let us assume that $S \in \Sigma_{L,N}$ and the number of the points $n = n(T)$ in the model*(9) *satisfies* (6)*. Then, the procedure S^* satisfies, for any $T \ge 32$, the following inequality*

$$\mathcal{R}(S^*, S) \le \frac{(1+\rho)^2(1+6\rho)}{1-6\rho} \min_{\alpha \in \mathcal{A}_\varepsilon} \mathcal{R}(S^*_\alpha, S) + \frac{\mathcal{B}_T(\rho)}{n},$$

where $\lim_{T\to\infty} \mathcal{B}_T(\rho)/n(T) = 0$.

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References

1. Karatzas I., Shreve S.E. *Methods of Mathematical Finance.* New York, Springer, 1998. xv, 415 p. doi: 10.1007/978-1-4939-6845-9.
2. Kutoyants Yu.A. *Statistical Inference for Ergodic Diffusion Processes.* London, Springer, 2004. xiv, 482 p. doi: 10.1007/978-1-4471-3866-2.
3. Galtchouk L.I., Konev V.V. On sequential estimation of parameters in continuous-time stochastic regression. *Statistics and Control of Stochastic Processes.* River Edge, World Sci. Publ., 1997, pp. 123–138.
4. Galtchouk L.I., Konev V.V. On sequential estimation of parameters in semimartingale regression models with continuous time parameter. *Ann. Stat.*, 2001, vol. 29, pp. 1508–2035.

5. Konev V.V., Pergamenshchikov S.M. On truncated sequential estimation of the parameters of diffusion processes. In: *Metody ekonomicheskogo analiza* [Methods of Economical Analysis]. Moscow, Tsentr. Ekon.-Mat. Inst. Ross. Akad. Nauk, 1992, pp. 3–31. (In Russian)

6. Galtchouk L.I., Pergamenshchikov S.M. Asymptotically efficient sequential kernel estimates of the drift coefficient in ergodic diffusion processes. *Stat. Inference Stoch. Processes*, 2006, vol. 9, no. 1, pp. 1–16. doi: 10.1007/s11203-005-3248-4.

7. Pchelintsev E. Improved estimation in a non-Gaussian parametric regression. *Stat. Inference Stoch. Processes*, 2013, vol. 16, no. 1, pp. 15–28. doi: 10.1007/s11203-013-9075-0.

8. Konev V.V, Pergamenshchikov S.M, Pchelintsev E.A. Estimation of a regression with the pulse type noise from discrete data. *Theory Probab. Its Appl.*, 2014, vol. 58, no. 3, pp. 442–457. doi: 10.1137/S0040585X9798662X.

9. Pchelintsev E., Pchelintsev V., Pergamenshchikov S. Improved robust model selection methods for the Lévy nonparametric regression in continuous time. *arXiv:1710.03111* , 2017, pp. 1–32. Available at: https://arxiv.org/submit/2029866.

10. Galtchouk L.I., Pergamenshchikov S.M. Adaptive sequential estimation for ergodic diffusion processes in quadratic metric. Part 1. Sharp non-asymptotic oracle inequalities. HAL Id hal-00177875, pp. 1–34. Available at: https://hal.archives-ouvertes.fr/hal-00177875.

11. Ibragimov I.A., Has'minskii R.Z. *Statistical Estimation: Asymptotic Theory.* New York, Springer, 1981. vii, 403 p. doi: 10.1007/978-1-4899-0027-2.

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Об улучшенном оценивании функции сноса в диффузионных процессах

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Аннотация

В работе рассмотрена задача робастного адаптивного непараметрического оценивания коэффициента сноса в диффузионных процессах. На основе последовательного подхода показано, что исходную задачу оценивания можно свести к задаче оценивания функции в дискретной непараметрической гетероскедастичной регрессионной модели. Предложена адаптивная процедура выбора модели на основе улучшенных взвешенных оценок по методу наименьших квадратов (МНК). Установлено, что такая оценка имеет более высокую неасимптотическую среднеквадратическую точность, чем процедура, построенная на основе классических взвешенных оценках МНК. Получено точное оракульное неравенство для квадратического риска предложенной процедуры оценивания, которое дает неасимптотическую верхнюю границу для риска.

Ключевые слова: улучшенное оценивание, стохастический диффузионный процесс, среднеквадратическая точность, выбор модели, оракульное неравенство

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