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On Solvability of a Fluid Flow Alpha-Model With Memory

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Abstract—We study weak solvability of an alpha-model in non-Newtonian hydrodynamics. If the parameter alpha equals zero, then the alpha-model coincides with the classical one describing a fluid flow with memory. This model takes into account fluid's memory along trajectories of movement. Additionally, we show that solutions of the alpha-model tend to solutions to the classical model as the parameter alpha tends to zero.

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In recent years, alpha-models in hydrodynamics are intensively investigated. In contrast to classical models, they are regularized approximate systems depending on a positive parameter α. As a rule, the systems are considered as independent correctly defined systems of equations. Because of this, their investigations is much simpler from both the analytic and the numeric points of view. Connection of alpha-models with classical ones, as α tends to zero, is of particular interest.

We should note that most part of papers on alpha-models is devoted to the ideal or Newtonial fluid. But in last time there are some papers on alpha-models for non-Newtonial fluids [1, 2]. The paper continues investigations in this direction. Its main aim is to investigate weak solvability of an alpha-model of viscous fluid flow with memory and to study behavior of weak solutions as $\alpha \to 0$.

Let Ω be a bounded domain in $\mathbb{R}^n$, $n = 2, 3$, with sufficiently smooth boundary $\partial\Omega$. In $Q_T = [0, T] \times \Omega$ we consider the following initial-boundary-value problem:

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} u_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - \mu_1 \mathrm{Div} \int_0^t e^{(s-t)/\lambda} \mathcal{E}(v)(s, z(s;t,x))\, ds + \nabla p = f, \quad (t,x) \in Q_T; \tag{1}$$

$$z(\tau; t, x) = x + \int_t^\tau v(s, z(s;t,x))\, ds, \quad (t,x) \in Q_T, \quad \tau \in [0, T]; \tag{2}$$

$$v = (I - \alpha^2 \Delta) u, \quad (t,x) \in Q_T; \quad \mathrm{div}\, v(t,x) = 0, \quad (t,x) \in Q_T; \tag{3}$$

$$v|_{[0,T]\times\partial\Omega} = 0, \quad v|_{t=0} = a, \quad x \in \Omega. \tag{4}$$

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