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EVALUATION OF NON-UNITAL QUBIT CHANNEL CAPACITIES

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Abstract

We have applied quantum Sinkhorn's theorem to non-unital qubit channels and derived lower and upper bounds on the classical capacity of such channels.

Keywords: qubit channel, non-unital channel, Holevo capacity

Introduction

Transmission of classical information through quantum channels has been covered in a number of papers [1–5] and reviews [6, 7]. In brief, if $R \in [0,1]$ is an achievable rate of information transmission, then n qubits effectively allow to transmit 2^{nR} classical messages.

The encoder assigns an n-qubit density operator $\varrho_i^{(n)}$ to each message i. The n-qubit density operator is a positive semidefinite operator with unit trace, which acts on 2^n dimensional Hilbert space $\mathcal{H}_{2^n}$. In the process of information transmission, each qubit is transmitted through a quantum channel Φ, which is a completely positive and trace preserving map. Therefore, the output state of n qubits reads $\Phi^{\otimes n}[\varrho_i^{(n)}]$. The decoder is a measurement device described by a positive operator-valued measure, which assigns a positive-semidefinite operator $M_j^{(n)}$ (acting on 2^n-dimensional Hilbert space) to each observed outcome $j \in \{1,\ldots,N\}$. Let $p(j|i)$ be the probability of observing outcome $j \in \{0,1,\ldots,N\}$ if the original message is i, then by the quantum-mechanical rule

$$p^{(n)}(j|i) = \mathrm{tr}[\varrho_i^{(n)} M_j^{(n)}].$$

Condition $\sum\limits_{j=1}^{N} M_j^{(n)} = I$ guarantees $\sum\limits_{j=1}^{N} p^{(n)}(j|i) = 1$. The maximum confusion probability reads

$$p_{\mathrm{err}}(n,N) = \max_{j=1,\ldots,N} \Big(1 - p^{(n)}(j|j)\Big).$$

$R \in [0,1]$ is called an achievable rate of information transmission if

$$\lim_{n\to\infty} p_{\mathrm{err}}(n, 2^{nR}) = 0.$$

By the classical capacity $C(\Phi)$ of quantum channel Φ we understand the supremum of achievable rates:

$$C(\Phi) = \sup \Big\{ R : \lim_{n\to\infty} p_{\mathrm{err}}(n, 2^{nR}) = 0 \Big\}.$$

The celebrated result in quantum information theory is that

$$C(\Phi) = \lim_{n\to\infty} \frac{1}{n} C_\chi(\Phi^{\otimes n}), \tag{1}$$

where the quantity $C_\chi(\Psi)$ is expressed through all possible ensembles of density operators $\{p_k, \rho_k\}$ and the von Neumann entropy $S(\rho) = -\mathrm{tr}(\rho \log_2 \rho)$ by formula

$$C_\chi(\Psi) = \sup_{\{p_k,\rho_k\}} \left[S\left(\sum_k p_k \Phi[\rho_k] \right) - \sum_k p_k S(\Phi[\rho_k]) \right].$$

We will refer to $C_\chi(\Psi)$ as the Holevo capacity of quantum channel Ψ.

Calculation of classical capacity $C(\Phi)$ is complicated in general. In this paper, we find lower and upper bounds on the capacity of general qubit channels.

1. Relation between unital and non-unital qubit channels

Let A and B be two operators acting on $\mathcal{H}_2$. By Φ_A we denote a completely positive map $\Phi_A[X] = AXA^\dagger$, i.e., a map with a single Kraus operator A. Analogously, $\Phi_B[X] = BXB^\dagger$. Hereafter, $\dagger$ denotes the Hermitian conjugation.

Suppose that Φ is a qubit map, which belongs to the interior of the cone of positivity preserving maps. Then, [8] states that there exist positive definite operators A and B acting on $\mathcal{H}_2$, such that the map

$$\Upsilon = \Phi_A \circ \Phi \circ \Phi_B \tag{2}$$

is unital, i.e., $\Upsilon(I) = I$, the identity operator. This result was also anticipated earlier as a quantum Sinkhorn's theorem [9]. In addition, if Φ is completely positive and trace preserving, then Υ is completely positive and trace preserving too. For the given non-unital qubit channel Φ, the particular form of operators A and B is derived in [10, 11]. Since A and B are nondegenerate, formula (2) implies that

$$\Phi = \Phi_{A^{-1}} \circ \Upsilon \circ \Phi_{B^{-1}},$$

i.e., all non-boundary non-unital qubit channels Φ can be decomposed into a concatenation of three completely positive maps $\Phi_{B^{-1}}$, Υ, $\Phi_{A^{-1}}$, with Υ being unital.

On the other hand, for any unital qubit channel Υ there exist unitary operators V and W, such that [12]

$$\Upsilon = \Phi_W \circ \Lambda \circ \Phi_V,$$

where the quantum channel Λ has a so-called diagonal form in the basis of conventional Pauli operators $I, \sigma_1, \sigma_2, \sigma_3$:

$$\Lambda[X] = \frac{1}{2}\mathrm{tr}[X] I + \frac{1}{2}\sum_{i=1}^{3} \lambda_i \mathrm{tr}[\sigma_i X]\sigma_i. \tag{3}$$

Parameters $\lambda_1, \lambda_2, \lambda_3$ in (3) are real and satisfy the constraint $1 \pm \lambda_3 \geqslant |\lambda_1 \pm \lambda_2|$ as Λ is completely positive [12].

Clearly, the classical capacities of channels Υ and Λ coincide. Moreover, since the additivity hypothesis holds true for unital qubit channels [13], the classical capacity equals the Holevo capacity and reads

$$C(\Upsilon) = C(\Lambda) = C_\chi(\Lambda) = 1 - h\left(\frac{1}{2}\Big(1 - \max_{i=1,2,3} |\lambda_i|\Big) \right), \tag{4}$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$.

In what follows, we relate the classical capacity of non-unital qubit channel Φ with the classical capacity of unital qubit channel Υ, which is given by formula (4).

2. Bounds on the classical capacity of non-unital qubit channels

Proposition 1. *Let us suppose that* Φ *is a non-unital qubit channel, such that the qubit map* $\Upsilon = \Phi_A \circ \Phi \circ \Phi_B$ *is unital. Then,* $C(\Phi) \geqslant C(\Upsilon) - 2\log_2(\|A\|\|B\|)$.

Proof. Let $\{\varrho_i^{(n)}, M_i^{(n)}\}_{i=1}^N$ be the optimal code of size $N = 2^{nR_\Upsilon}$ for the composite channel $\Upsilon^{\otimes n}$ such that $\lim_{n\to\infty} p_{\rm err\ \Upsilon}(n, 2^{nR_\Upsilon}) = 0$.

Consider a set of modified input states

$$\widetilde{\varrho}_i^{(n)} = \frac{B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}}{{\rm tr}[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}]}$$

and a modified positive operator-valued measure $\{j \to \widetilde{M}_j^{(n)}\}_{j=0}^N$ with elements

$$\widetilde{M}_0^{(n)} = I - \sum_{j=1}^N \widetilde{M}_j^{(n)}, \quad \widetilde{M}_j^{(n)} = \frac{(A^\dagger)^{\otimes n} M_j^{(n)} A^{\otimes n}}{\|A\|^{2n}}, \quad j = 1, \ldots, N,$$

where $\|X\| = \|X\|_\infty = \max_{\psi:\langle\psi|\psi\rangle=1} \langle\psi|X^\dagger X|\psi\rangle$ is the operator norm. It is not hard to see that $\widetilde{M}_0^{(n)}$ is positive semidefinite.

Using the modified code, let each qubit be transmitted through the channel Φ. Then, the probability to observe outcome $j \neq 0$ provided input message i equals

$$\widetilde{p}^{(n)}(j|i) = {\rm tr}\left[\widetilde{\varrho}_i^{(n)}\widetilde{M}_j^{(n)}\right] = \frac{{\rm tr}\left\{A^{\otimes n}\Phi^{\otimes n}\left[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}\right](A^\dagger)^{\otimes n}M_j^{(n)}\right\}}{{\rm tr}[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}]\|A\|^{2n}}.$$

Since $\Phi_A \circ \Phi \circ \Phi_B = \Upsilon$, we get

$$\widetilde{p}^{(n)}(j|i) = \frac{{\rm tr}\left\{\Upsilon^{\otimes n}[\varrho_i^{(n)}]M_j^{(n)}\right\}}{{\rm tr}[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}]\|A\|^{2n}} = \frac{p^{(n)}(j|i)}{{\rm tr}[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}]\|A\|^{2n}},$$

where $p^{(n)}(j|i)$ is the probability to get outcome $j \in \{1, \ldots, N\}$ for the input message $i \in \{1, \ldots, N\}$ in the original optimal protocol for channel $\Upsilon^{\otimes n}$.

Observation of the outcome $j = 0$ in the modified protocol would be treated as an unsuccessful event, whereas observation of the outcome $j \in \{1, \ldots, N\}$ leads to a successful identification of the message because $p^{(n)}(j|i) \to \delta_{ij}$ if $n \to \infty$.

The probability to observe nonzero outcome j equals

$$P^{(n)} = \sum_{j=1}^N \widetilde{p}^{(n)}(j|i) = \frac{1}{{\rm tr}[B^{\otimes n}\varrho_i^{(n)}(B^\dagger)^{\otimes n}]\|A\|^{2n}} \geqslant \frac{1}{(\|A\|\,\|B\|)^{2n}}.$$

Utilizing the modified protocol, one can transmit information only in the case of successful events $j \neq 0$, so the average number of successfully transmitted messages $\widetilde{N}$ equals

$$\widetilde{N} = P^{(n)}N = P^{(n)}2^{nR_\Upsilon} \geqslant 2^{n(R_\Upsilon - 2\log_2(\|A\|\,\|B\|))}.$$

Therefore, the considered protocol enables one to achieve the rate

$$\widetilde{R} \geqslant R_\Upsilon - 2\log_2(\|A\|\|B\|) \tag{5}$$

by utilizing the channel Φ.

If $R_\Upsilon \leqslant C(\Upsilon)$ and one observes the successful event ($j \neq 0$), then the maximum error probability in the modified protocol

$$\widetilde{p}_{\rm err}(n, \widetilde{N}) = \max_{j=1,\ldots,N} \left(1 - \frac{\widetilde{p}^{(n)}(j|j)}{P^{(n)}}\right) = \max_{j=1,\ldots,N} \left(1 - p^{(n)}(j|j)\right) \to 0 \quad \text{if} \quad n \to \infty.$$

Taking the supremum on both sides of eq. (5) with requirement $\lim_{n\to\infty} \widetilde{p}_{\rm err}(n, \widetilde{N}) = 0$, we get

$$C(\Phi) \geqslant C(\Upsilon) - 2\log_2(\|A\|\|B\|).$$

□

In the proof of proposition 1, we have used only the relation $\Upsilon = \Phi_A \circ \Phi \circ \Phi_B$. Instead, if we use the relation $\Phi = \Phi_{A^{-1}} \circ \Upsilon \circ \Phi_{B^{-1}}$, we immediately get $C(\Upsilon) \geqslant C(\Phi) - 2\log_2(\|A^{-1}\|\|B^{-1}\|)$. Therefore, we immediately obtain the upper bound on capacity $C(\Phi)$.

Proposition 2. *Let us suppose that Φ is non-unital qubit channel and its decomposition through the unital qubit channel Υ reads $\Phi = \Phi_{A^{-1}} \circ \Upsilon \circ \Phi_{B^{-1}}$. Then, $C(\Phi) \leqslant C(\Upsilon) + 2\log_2(\|A^{-1}\|\|B^{-1}\|)$.*

Combining propositions 1 and 2, we get the following result.

Corollary 1. *Let Φ be a unital qubit channel belonging to the interior of positive qubit maps, then there exist positive definite operators A and B acting on $\mathcal{H}_2$, such that the map $\Upsilon = \Phi_A \circ \Phi \circ \Phi_B$ is unital and*

$$C(\Upsilon) - 2\log_2(\|A\|\|B\|) \leqslant C(\Phi) \leqslant C(\Upsilon) + 2\log_2(\|A^{-1}\|\|B^{-1}\|).$$

Proof. The statement straightforwardly follows from the decomposition existence [8] and propositions 1 and 2. □

3. Four-parameter non-unital qubit channels

Consider a non-unital qubit channel of the form

$$\Phi[X] = \frac{1}{2}\left(\mathrm{tr}[X](I + t_3\sigma_3) + \sum_{j=1}^{3} \lambda_j \mathrm{tr}[\sigma_j \varrho]\sigma_j\right),$$

where t_3 and $\lambda_1, \lambda_2, \lambda_3$ are real parameters, which in addition to the condition of complete positivity also satisfy the inequality that $|t_3| + |\lambda_3| < 1$. It guarantees that Φ is an interior point of the cone of positive qubit maps. In [11], the explicit form of decomposition $\Phi = \Phi_{A^{-1}} \circ \Upsilon \circ \Phi_{B^{-1}}$ is provided:

$$A = \frac{2}{\sqrt{(1+t_3)^2 - \lambda_3^2} + \sqrt{(1-t_3)^2 - \lambda_3^2}} \begin{pmatrix} \sqrt{(1+|t_3|)^2 - \lambda_3^2} & 0 \\ 0 & \sqrt{(1-|t_3|)^2 - \lambda_3^2} \end{pmatrix},$$

$$B = \begin{pmatrix} \dfrac{1}{\sqrt{1 + t_3x_3 + |\lambda_3 x_3|}} & 0 \\ 0 & \dfrac{1}{\sqrt{1 + t_3x_3 - |\lambda_3 x_3|}} \end{pmatrix},$$

$$x_3 = -t_3 \frac{1 - t_3^2 + \lambda_3^2 + \sqrt{[(1+t_3)^2 - \lambda_3^2][(1-t_3)^2 - \lambda_3^2]}}{1 - t_3^2 - \lambda_3^2 + \sqrt{[(1+t_3)^2 - \lambda_3^2][(1-t_3)^2 - \lambda_3^2]}},$$

and the unital qubit map $\Upsilon = \widetilde{\Lambda}$ has the form (3) with parameters

$$\widetilde{\lambda}_1 = \frac{2\lambda_1}{\sqrt{(1+\lambda_3)^2 - t_3^2} + \sqrt{(1-\lambda_3)^2 - t_3^2}}, \quad (6)$$

$$\widetilde{\lambda}_2 = \frac{2\lambda_2}{\sqrt{(1+\lambda_3)^2 - t_3^2} + \sqrt{(1-\lambda_3)^2 - t_3^2}}, \quad (7)$$

$$\widetilde{\lambda}_3 = \frac{4\lambda_3}{\left(\sqrt{(1+\lambda_3)^2 - t_3^2} + \sqrt{(1-\lambda_3)^2 - t_3^2}\right)^2}. \quad (8)$$

We explicitly find the operator norms

$$\|A\| = \frac{2}{1 + \sqrt{\dfrac{(1-|t_3|)^2 - \lambda_3^2}{(1+|t_3|)^2 - \lambda_3^2}}}, \quad (9)$$

$$\|A^{-1}\| = \frac{1}{2}\left(1 + \sqrt{\frac{(1+|t_3|)^2 - \lambda_3^2}{(1-|t_3|)^2 - \lambda_3^2}}\right), \quad (10)$$

$$\|B\| = \frac{1}{\sqrt{1 + t_3 x_3 - |\lambda_3 x_3|}}, \quad (11)$$

$$\|B^{-1}\| = \sqrt{1 + t_3 x_3 + |\lambda_3 x_3|}. \quad (12)$$

By substituting these norms in corollary 1, we find the lower and upper bounds on capacity $C(\Phi)$.

Proposition 3. *The classical capacity of non-unital qubit channel*

$$\Phi[X] = \frac{1}{2}\mathrm{tr}[X](I + t_3\sigma_3) + \frac{1}{2}\sum_{j=1}^{3} \lambda_j \mathrm{tr}[\sigma_j \varrho]\sigma_j$$

with $|t_3| + |\lambda_3| < 1$ *satisfies*

$$C(\Phi) \geqslant 1 - h\left(\frac{1}{2}\Big(1 - \max_{i=1,2,3} |\widetilde{\lambda}_i|\Big)\right) - 2\log_2(\|A\|\,\|B\|),$$

$$C(\Phi) \leqslant 1 - h\left(\frac{1}{2}\Big(1 - \max_{i=1,2,3} |\widetilde{\lambda}_i|\Big)\right) + 2\log_2(\|A^{-1}\|\,\|B^{-1}\|),$$

where $\widetilde{\lambda}_i$, $i = 1,2,3$ *are given by formulas* (6)–(8) *and* $\|A\|$, $\|A^{-1}\|$, $\|B\|$, $\|B^{-1}\|$ *are given by formulas* (9)–(12).

Conclusions

We have obtained new lower and upper bounds on the classical capacities of non-unital qubit channels. We must emphasize that the obtained result holds true for the regularized version of the Holevo capacity, formula (1). Our proofs are based on the seminal relation between unital and non-unital qubit channels, which was developed in [8] and [11]. This relation can be productive in other research areas as well, for instance, in the study of divisibility of qubit dynamical maps [14–18] and their tensor products [19], in the study of entanglement annihilation [20–24] and absolutely separating quantum channels [25], in the study of quantum capacities and other types of capacities [6, 7]. For practical applications, we have derived lower and upper bounds for a four-parameter family of non-unital qubit channels. For instance, this family covers generalized amplitude damping channels, which originate from the processes of emission and absorption due to interaction of the two-level system (qubit) with a reservoir of finite temperature [26].

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Анализ пропускной способности неунитальных кубитных каналов

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Аннотация

Квантовая теорема Синкхорна применяется к неунитальных кубитным каналам. Находятся верхняя и нижняя границы для классической пропускной способности таких каналов.

Ключевые слова: кубитный канал, неунитальный канал, пропускная способность Холево

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