Determination of cardinal points of complex optical systems.

<u>Experiment Objective:</u> determination of the main focal lengths and principal plane position of centered optical system consisting of 2 positive lenses.

<u>Tasks:</u>

- to acquire skills of the centered optical systems adjustment;
- to learn the methods of measuring the converging lenses focal length;
- to investigate experimentally the dependence of the image view on the object position relatively to the lens focal length;
- master the method of complex optical systems analysis

<u>Optical elements and apparatus:</u>

- ✓ optical rail with supports (riders);
- ✓ incandescent electric lamp in case with power supply;
- ✓ aspherical condenser with a tripod for the diaphragm;
- ✓ translucent screen;
- ✓ 2 converging lenses with focal lengths f = 150 mm, 200 mm;
- ✓ arrow slit diaphragm;
- ✓ tape measure 2 m long.

<u>Procedure:</u>

1. Mount the incandescent electric lamp, supports with the first lens and translucent screen to the optical rail. Inserted the arrow slit diaphragm in the rider fixed to the lamp housing. Determine the main focal length of the first lens. Get the image of the object (arrow) on the screen and measure the distance from the lens to the object ($-a_1$) and from the lens to the image (a_2). Repeat these measurements 6 - 8 times for various positions of the lens. Since the distances $-a_1$ and a_2 are related as $1/a_2 - 1/a_1 = 1/f_2$, the focal length of the first lens f_2 can be determined by plotting the dependence $1/a_2$ on $1/(-a_1)$.
2. Similarly, determine the focal length of the second thin lens.
3. Mount on the optical rail both optical lenses so that their holders were closely fitting with each other. The distance between the lenses will be ~ 6.5 cm. Calculate the mail focal length F of the optical system by the Bessel's method. To do this, set the distance L between the object and the screen 70 - 80 cm. Shift the system along the optical bench until the enlarged image will be observed on the screen. Measure the distance from the object to the first lens ($-a_1$), Fig. 1a. Then shift the system along the optical bench until the reduced image will be observed on the screen. Measure the distance again from the object to the first lens ($-a_2$), Fig. 1b. Determine the distance S on which system was shifted: $S = (-a_2) - (-a_1)$.
4. Determine the focal lengths of the system by Bessel's formula:

$$F_2 = -F_1 = \frac{(L-e)^2 - S^2}{4(L-e)}. \qquad (1)$$

where *e* – distance between principal planes *H* and *H'*. This value can be calculated by a matrix method for example. Derivation of formula (1) is given in Appendix.

5. Determine the principal planes and focus position of a system consisting of two positive lenses with focal lengths which were calculated previously in exercises 1 and 2 by plotting.

Appendix

The derivation of Bessel's formula for complex optical systems.

An optical system (OS) project a real image on a screen (Fig.1a). Let *L* - distance between the object and image, ($-a_1$) – distance from object to principal plane H, a_1' – distance from principal plane H' to image, *e* – distance between principal planes.

Then $L = a_1' + e - a_1$, so $a_1' = (L - e) + a_1$.

In our case $\frac{1}{a_1} - \frac{1}{a_1'} = -\frac{1}{F_2}$

Substituting the a_1' in the last expression the quadratic equation will obtain:

$$a_1^2 + (L-e)a_1 + (L-e)F_2 = 0. \qquad (2)$$

with a discriminant $L = \sqrt{(L-e)^2 - 4F_2(L-e)}$.

If $(L - e) > 4F$ then the equation has two real roots

$$a_1 = \frac{-(L-e)+\sqrt{D}}{2} \text{ and } a_2 = \frac{-(L-e)-\sqrt{D}}{2}$$

This means that there are two positions of the optical system (OS), one of which corresponds to an enlarged (Fig. 1a), and another to a reduced (Fig. 1b) object image. The distance between the object and the screen (*L*) is constant.

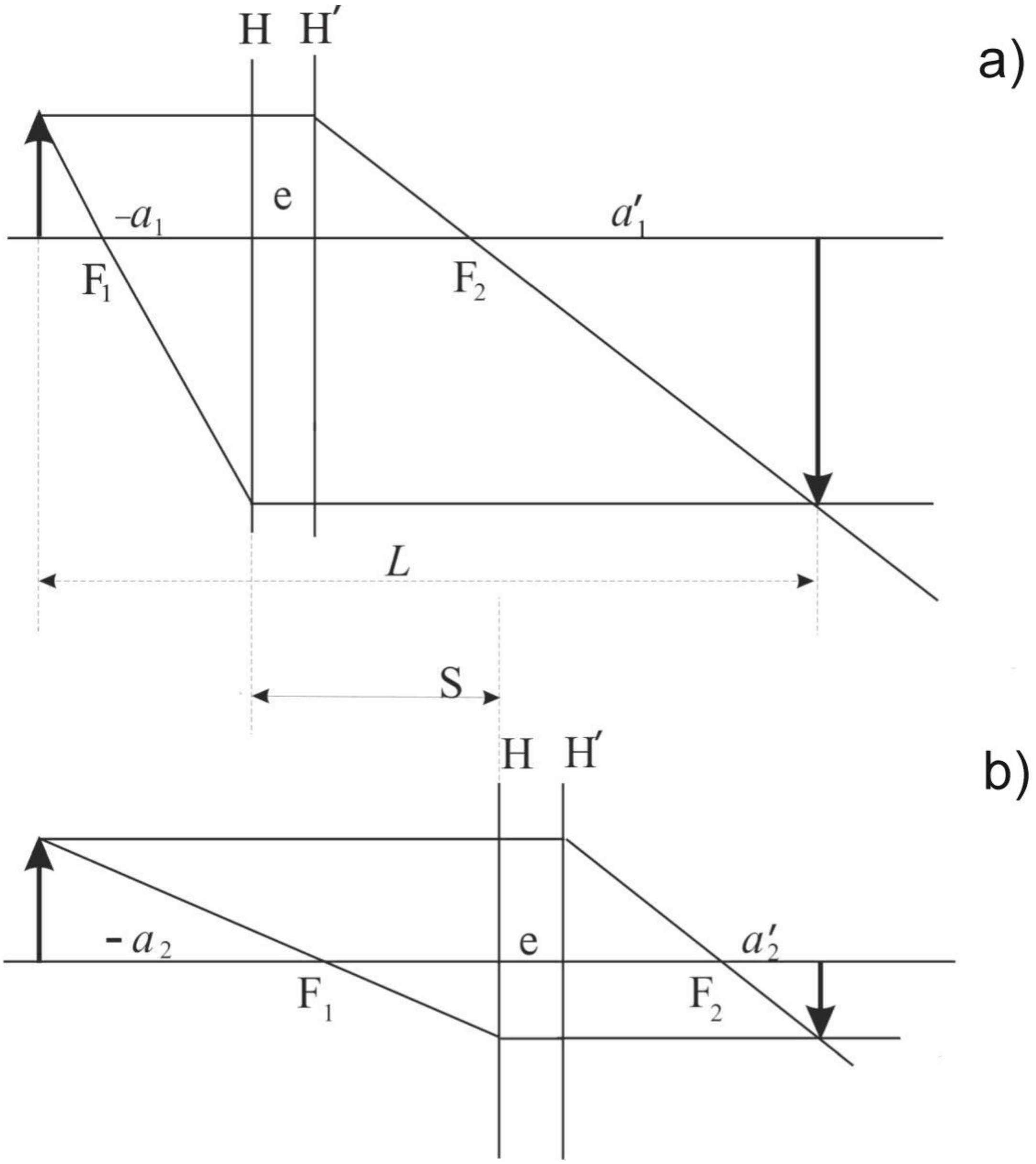

Fig. 1. Schematic sketch of the geometry for Bessel's formula derivation.

As a_1 and a_2 are negative values and $|a_2| > |a_1|$, the positive value S which is the distance between the front principal planes for the first and second positions (see Fig.1):

$$S = a_1 - a_2 = \sqrt{(L-e)^2 - 4F_2(L-e)}. \tag{3}$$

Hence $F_2 = \frac{(L-e)^2 - S^2}{4(L-e)} = -F_1$.