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On Algebras Over Multicategories

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Abstract—We introduce a notion of a monoidal category over verbal category. In such categories we define algebras over multicategories over the same verbal categories. We also explicitly compute categories of algebras for two classes of multicategories.

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INTRODUCTION

Here we continue the study initiated in [1–4]. The main object of consideration is multigraded (color) operads, also called multicategories. The author has already introduced the sweeping generalization of multicategories, namely, multicategories over verbal categories. In order to study the algebra categories over these multicategories in more general than set-theoretical cases we introduce one new class of monoidal (tensor) categories, i.e., a monoidal categories over a verbal algebras. This allows us to define a notion of algebra over a multicategory over a verbal category as the object of a monoidal category.

The paper consists of three Sections. The first Section introduces the notion of the monoidal category over the verbal category. We also point out certain links with the double category theory and give a number of examples. In the second Section we show that the known functor that maps a monoidal category over a verbal category into multicategory in fact constructs a multicategory over the relative verbal category. Then we introduce the notion of an algebra over a multicategory over a verbal category as a multifunctor from the given multicategory into the already mentioned multicategory constructed by the monoidal category. Finally, in the third Section we apply the newly-built notation for the explicit calculation of the algebra categories over the two naturally defined multicategory classes (matrix and semigroup algebra operads generalizations of [5]).

The notation follows the authors works [1, 3, 4].

1. MONOIDAL CATEGORIES OVER VERBAL CATEGORIES

We start with the lemma, whose formulation in compressed form (in double categories terms [6]) actually contain multicategories over verbal categories definition. Throughout the paper the top line denotes finite ordered index sequence, most often the numbered letter set, for example, $\overline{x} = x_1x_2\dots x_n$. The other names of such sequences are strings or words in some alphabet. Recall also the notion $[n] = \{0, 1, \dots, n\}$. Verbal categories then are categories with objects of type $[n]$. The morphisms of these categories are mappings of the form $[n] \to [m]$, under which only zero and no other element maps into zero. All the mappings with such a property are morphisms of the verbal category FSet. The other important example of a verbal category is the category Σ whose morphisms are the bijections of type $[n] \to [n]$ (of the same property with respect to zero). Moreover, $\Sigma([n], [n]) = \Sigma_n$ is the substitution group of degree n. The exact definition of the verbal categories can be found in [2] and [4]. Further on we recall certain properties of the verbal categories.

After we compare the multicategories over verbal categories ([1], definition 3) and double categories definitions we obtain

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