

ТИПОВЫЕ ЗАДАНИЯ ПО ДИСЦИПЛИНЕ «МАТЕМАТИКА-1»

1. Вычислить определитель:

а) по правилу треугольников; б) разложением по какой-нибудь строке или столбцу.

$$1.1 \begin{vmatrix} -1 & 5 & 1 \\ -2 & 6 & 2 \\ 2 & 7 & 3 \end{vmatrix} \cdot 1.2 \begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 3 \\ -1 & -3 & 4 \end{vmatrix} \cdot 1.3 \begin{vmatrix} 6 & 10 & 6 \\ 8 & 8 & 5 \\ 5 & 9 & 5 \end{vmatrix} \cdot 1.4 \begin{vmatrix} 7 & 9 & 7 \\ 6 & 8 & 6 \\ -7 & 9 & 7 \end{vmatrix} \cdot 1.5 \begin{vmatrix} 6 & 8 & -9 \\ 4 & 6 & -6 \\ -2 & -3 & 4 \end{vmatrix}.$$

$$1.6 \begin{vmatrix} 2 & -5 & 1 \\ -3 & 7 & -1 \\ 5 & -9 & 2 \end{vmatrix} \cdot 1.7 \begin{vmatrix} 9 & 3 & 6 \\ 8 & 2 & 7 \\ 5 & 3 & 2 \end{vmatrix} \cdot 1.8 \begin{vmatrix} 2 & 4 & 3 \\ 3 & 7 & 5 \\ 4 & 8 & 5 \end{vmatrix} \cdot 1.9 \begin{vmatrix} 3 & -5 & 2 \\ -3 & 4 & -5 \\ 8 & -8 & 5 \end{vmatrix} \cdot 1.10 \begin{vmatrix} 7 & 6 & 3 \\ 3 & 5 & 7 \\ 5 & 4 & 3 \end{vmatrix}.$$

$$1.11 \begin{vmatrix} 1 & 6 & 5 \\ 2 & 8 & 6 \\ 4 & 9 & 3 \end{vmatrix} \cdot 1.12 \begin{vmatrix} 3 & -3 & 6 \\ 2 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix} \cdot 1.13 \begin{vmatrix} 10 & 6 & 6 \\ 8 & 8 & 5 \\ 9 & 5 & 5 \end{vmatrix} \cdot 1.14 \begin{vmatrix} -6 & 8 & 6 \\ 6 & 8 & 6 \\ 7 & 9 & 7 \end{vmatrix} \cdot 1.15 \begin{vmatrix} 8 & 6 & -9 \\ 6 & 4 & -6 \\ -3 & -2 & 4 \end{vmatrix}.$$

$$1.16 \begin{vmatrix} 2 & -5 & 1 \\ 4 & 7 & -1 \\ 2 & -6 & 1 \end{vmatrix} \cdot 1.17 \begin{vmatrix} 6 & 9 & 3 \\ 7 & 8 & 2 \\ 5 & 8 & 4 \end{vmatrix} \cdot 1.18 \begin{vmatrix} 4 & 2 & 3 \\ 7 & 3 & 5 \\ 8 & 4 & 5 \end{vmatrix} \cdot 1.19 \begin{vmatrix} -5 & 3 & 2 \\ 4 & -3 & -5 \\ -8 & 8 & 5 \end{vmatrix} \cdot 1.20 \begin{vmatrix} 7 & 6 & 3 \\ 2 & 5 & 7 \\ 5 & 4 & 3 \end{vmatrix}.$$

$$1.21 \begin{vmatrix} 4 & 9 & 3 \\ 2 & 8 & 6 \\ 1 & 6 & 5 \end{vmatrix} \cdot 1.22 \begin{vmatrix} 6 & -3 & 3 \\ 3 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} \cdot 1.23 \begin{vmatrix} 7 & 7 & 4 \\ 8 & 8 & 5 \\ 9 & 5 & 5 \end{vmatrix} \cdot 1.24 \begin{vmatrix} 6 & 8 & -8 \\ 6 & 8 & 8 \\ 7 & 9 & 9 \end{vmatrix} \cdot 1.25 \begin{vmatrix} -3 & 4 & 6 \\ -4 & 6 & 8 \\ 8 & -9 & 12 \end{vmatrix}.$$

$$1.26 \begin{vmatrix} 7 & 2 & 5 \\ 2 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} \cdot 1.27 \begin{vmatrix} 5 & 8 & 4 \\ 7 & 8 & 2 \\ 2 & 5 & 3 \end{vmatrix} \cdot 1.28 \begin{vmatrix} 8 & 4 & 5 \\ 7 & 3 & 5 \\ 4 & 2 & 3 \end{vmatrix} \cdot 1.29 \begin{vmatrix} 7 & -5 & 5 \\ 4 & -3 & 3 \\ -5 & 3 & -4 \end{vmatrix} \cdot 1.30 \begin{vmatrix} 5 & 3 & 5 \\ 2 & 7 & 3 \\ 4 & 5 & 5 \end{vmatrix}.$$

2. Даны матрицы A, B . Найти матрицы C, D .

$$2.1. A = \begin{pmatrix} -4 & 5 & 1 \\ 2 & -3 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} -7 & 4 \\ 10 & -5 \\ 3 & 16 \end{pmatrix}, \quad C = B - 2A^T, \quad D = A \cdot B$$

$$2.2. A = \begin{pmatrix} 4 & 1 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}, \quad C = (3A)^T - B, \quad D = A \cdot B$$

$$2.3. A = \begin{pmatrix} 5 & -2 \\ 4 & 3 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -4 & -1 & 5 \\ 2 & 3 & -1 \end{pmatrix}, \quad C = A + B^T, \quad D = A \cdot B$$

$$2.4. A = \begin{pmatrix} 2 & 3 & 5 \\ -1 & 2 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 & -1 \\ 4 & 3 & 2 \end{pmatrix}, \quad C = 2A + 3B, \quad D = A \cdot (B^T)$$

$$2.5. A = \begin{pmatrix} 3 & 0 \\ -2 & 1 \\ 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 1 \\ -3 & 1 \\ 2 & 1 \end{pmatrix}, \quad C = A^T - B^T, \quad D = A^T \cdot B$$

$$2.6. A = \begin{pmatrix} 2 & -4 \\ 3 & 7 \\ 6 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 0 & 1 \\ 6 & 1 & -3 \end{pmatrix}, \quad C = 2A - B^T, \quad D = A \cdot B$$

$$2.7. A = \begin{pmatrix} 3 & 1 & 0 \\ -2 & -1 & 4 \\ 7 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1 & 3 \\ 2 & -1 & 4 \\ 4 & 6 & 2 \end{pmatrix}, \quad C = (2B)^T + A, \quad D = A \cdot B$$

$$2.8. A = \begin{pmatrix} 3 & -4 & 2 \\ 7 & 5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 \\ 6 & 6 \\ -3 & 1 \end{pmatrix}, \quad C = A - B^T, \quad D = A \cdot B$$

$$2.9. A = \begin{pmatrix} 7 & -4 \\ 0 & -3 \\ 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 4 \\ -1 & 2 \\ 6 & 3 \end{pmatrix}, \quad C = 3B - 2A, \quad D = A^T \cdot B$$

$$2.10. A = \begin{pmatrix} 6 & 1 & -3 \\ 2 & -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 3 & 1 \\ -5 & 1 & 4 \end{pmatrix}, \quad C = A^T + B^T, \quad D = A \cdot (B^T)$$

$$2.11. A = \begin{pmatrix} -2 & 1 & -5 \\ 4 & 3 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 1 \\ 3 & 0 \\ -4 & -2 \end{pmatrix}, \quad C = 2B - A^T, \quad D = A \cdot B$$

$$2.12. A = \begin{pmatrix} -4 & 2 & -1 \\ -1 & 0 & -2 \\ 3 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -2 & 1 \\ 5 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}, \quad C = (2A)^T - B, \quad D = A \cdot B$$

$$2.13. A = \begin{pmatrix} -4 & 3 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -7 & 1 & -2 \\ 1 & 3 & 2 \end{pmatrix}, \quad C = 3A + B^T, \quad D = A \cdot B$$

$$2.14. A = \begin{pmatrix} -2 & 3 & 4 \\ -1 & 2 & -5 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 1 & 0 \\ -3 & 2 & 1 \end{pmatrix}, \quad C = 2A - 3B, \quad D = A \cdot (B^T)$$

$$2.15. A = \begin{pmatrix} -2 & 1 \\ 5 & -7 \\ 4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 1 \\ -2 & -1 \\ 3 & 0 \end{pmatrix}, \quad C = 2(A^T) - B^T, \quad D = A^T \cdot B$$

$$2.16. A = \begin{pmatrix} 4 & -3 & 2 \\ 1 & 0 & 3 \\ 9 & 7 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 & -2 \\ 4 & 3 & 2 \\ -8 & -3 & 2 \end{pmatrix}, \quad C = A + B^T, \quad D = A \cdot B$$

$$2.17. A = \begin{pmatrix} 3 & -1 \\ 6 & 8 \\ -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 1 & 0 \\ -7 & 1 & -2 \end{pmatrix}, \quad C = A - 2(B^T), \quad D = A \cdot B$$

$$2.18. A = \begin{pmatrix} 4 & 1 & -3 \\ 2 & 0 & 5 \\ 0 & -4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -7 & 8 & 1 \\ 0 & 3 & 0 \\ -2 & 1 & 1 \end{pmatrix}, \quad C = 3A - B, \quad D = A \cdot B$$

$$2.19. A = \begin{pmatrix} -2 & 1 & 4 \\ -3 & 7 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 \\ -3 & 2 \\ 0 & -3 \end{pmatrix}, \quad C = 2A + B^T, \quad D = A \cdot B$$

$$2.20. A = \begin{pmatrix} 9 & -11 \\ 7 & -3 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & -2 \\ 1 & -3 \\ 8 & -1 \end{pmatrix}, \quad C = A - 3B, \quad D = A^T \cdot B$$

$$2.21. A = \begin{pmatrix} -3 & 2 & 1 \\ -4 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 9 & -8 & 7 \\ -6 & 5 & -4 \end{pmatrix}, \quad C = A^T + 2(B^T), \quad D = A \cdot (B^T)$$

$$2.22. A = \begin{pmatrix} 3 & 4 & -2 \\ 1 & 0 & -3 \\ -7 & 5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 & 2 \\ -3 & 1 & 4 \\ 2 & -1 & 3 \end{pmatrix}, \quad C = A - B, \quad D = A \cdot B$$

$$2.23. A = \begin{pmatrix} -4 & 0 & 5 \\ -6 & -2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & -1 \\ 3 & 2 \\ 4 & -3 \end{pmatrix}, \quad C = 3B - A^T, \quad D = A \cdot B$$

$$2.24. A = \begin{pmatrix} 7 & -3 & 1 \\ 0 & 2 & 1 \\ -4 & 5 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 & 3 \\ 4 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \quad C = A + (2B)^T, \quad D = A \cdot B$$

$$2.25. A = \begin{pmatrix} -6 & 1 \\ 0 & -2 \\ 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 & -3 \\ -2 & 0 & 1 \end{pmatrix}, \quad C = 4A + B^T, \quad D = A \cdot B$$

$$2.26. A = \begin{pmatrix} -2 & 3 & 0 \\ -4 & -2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & -3 & 1 \\ -1 & 2 & 3 \end{pmatrix}, \quad C = 3A + 2B, \quad D = A \cdot (B^T)$$

$$2.27. A = \begin{pmatrix} -3 & 1 \\ 5 & -7 \\ 3 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 7 \\ -1 & -2 \\ 4 & 0 \end{pmatrix}, \quad C = A^T - 2(B^T), \quad D = A^T \cdot B$$

$$2.28. A = \begin{pmatrix} 3 & -4 & 0 \\ -2 & 0 & 1 \\ 6 & 7 & -5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 3 & 1 & 0 \end{pmatrix}, \quad C = A^T + B, \quad D = A \cdot B$$

$$2.29. A = \begin{pmatrix} -3 & 6 \\ 7 & 1 \\ -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 1 & 4 \\ -3 & 2 & 1 \end{pmatrix}, \quad C = A^T - 3B, \quad D = A \cdot B$$

$$2.30. A = \begin{pmatrix} -4 & 0 & 2 \\ -6 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 1 & 3 \\ -2 & 1 & 0 \\ 1 & -4 & 2 \end{pmatrix}, \quad C = 2A + B, \quad D = A \cdot B$$

3. Дана матрица A . Найдите: $|A|$, A^{-1} , $A \cdot A^{-1}$.

$$3.1. A = \begin{pmatrix} -2 & 3 & 1 \\ 3 & 6 & 2 \\ 1 & 2 & 1 \end{pmatrix}. \quad 3.2. A = \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}. \quad 3.3. A = \begin{pmatrix} 1 & -3 & -1 \\ -2 & 7 & 2 \\ 3 & 2 & -4 \end{pmatrix}.$$

$$3.4. A = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}. \quad 3.5. A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}. \quad 3.6. A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 1 & 0 & 4 \end{pmatrix}.$$

$$3.7. A = \begin{pmatrix} 4 & -8 & -5 \\ -4 & 7 & -1 \\ -3 & 5 & 1 \end{pmatrix}. \quad 3.8. A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 3 & -2 \\ -5 & -4 & -1 \end{pmatrix}. \quad 3.9. A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 1 \\ 4 & 1 & -5 \end{pmatrix}.$$

$$3.10. A = \begin{pmatrix} 2 & -1 & 0 \\ 5 & 3 & -6 \\ -1 & -2 & 3 \end{pmatrix}. \quad 3.11. A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}. \quad 3.12. A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}.$$

$$3.13. A = \begin{pmatrix} 1 & 2 & 0 \\ 4 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}. \quad 3.14. A = \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}. \quad 3.15. A = \begin{pmatrix} 4 & -8 & -5 \\ -4 & 7 & -1 \\ -3 & 5 & 1 \end{pmatrix}.$$

$$3.16. A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}. \quad 3.17. A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 4 & 3 \end{pmatrix}. \quad 3.18. A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & -2 & 1 \\ 2 & 2 & 3 \end{pmatrix}.$$

$$3.19. A = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 7 & 2 \\ -1 & 2 & -4 \end{pmatrix}. \quad 3.20. A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -3 & 2 & 4 \end{pmatrix}. \quad 3.21. A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & -2 \\ 2 & 1 & -1 \end{pmatrix}.$$

$$3.22. A = \begin{pmatrix} 5 & 3 & -6 \\ 2 & -1 & 0 \\ -1 & -2 & 3 \end{pmatrix}. \quad 3.23. A = \begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 2 \\ 3 & 0 & 1 \end{pmatrix}. \quad 3.24. A = \begin{pmatrix} -4 & 4 & 3 \\ -8 & 7 & 5 \\ 5 & 1 & -1 \end{pmatrix}.$$

$$3.25. A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{pmatrix}. \quad 3.26. A = \begin{pmatrix} 4 & 5 & 1 \\ 1 & 2 & 0 \\ 3 & 1 & 1 \end{pmatrix}. \quad 3.27. A = \begin{pmatrix} 2 & 3 & 1 \\ 7 & 9 & 5 \\ 3 & 4 & 3 \end{pmatrix}.$$

$$3.28. A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 3 \end{pmatrix}.$$

$$3.29. A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & -2 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

$$3.30. A = \begin{pmatrix} 3 & 2 & 3 \\ 4 & 3 & 5 \\ -5 & -1 & 1 \end{pmatrix}.$$

4. Решить систему линейных алгебраических уравнений тремя способами:

а) методом Крамера; б) методом Гаусса в) методом обратной матрицы.

$$4.1 \begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11 \end{cases}$$

$$4.2 \begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$$

$$4.3 \begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$$

$$4.4 \begin{cases} x_1 + x_2 + 2x_3 = 2 \\ 3x_1 + 2x_2 + x_3 = 2 \\ -x_1 - x_2 + 4x_3 = 4 \end{cases}$$

$$4.5 \begin{cases} x_1 + 5x_2 - 4x_3 = -5 \\ 2x_1 - 3x_2 + x_3 = 2 \\ 4x_1 + x_2 - 3x_3 = -4 \end{cases}$$

$$4.6 \begin{cases} x_1 - 2x_2 + 4x_3 = 3 \\ 2x_1 - 4x_2 + 3x_3 = 1 \\ 3x_1 - x_2 + 5x_3 = 2 \end{cases}$$

$$4.7 \begin{cases} 2x_1 - x_2 + x_3 = 2 \\ 3x_1 + 2x_2 + 2x_3 = -2 \\ x_1 - 2x_2 + x_3 = 1 \end{cases}$$

$$4.8 \begin{cases} x_1 + 3x_2 + 4x_3 = 6 \\ 2x_1 - x_2 - x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 5 \end{cases}$$

$$4.9 \begin{cases} x_1 + x_2 + x_3 = 3 \\ 2x_1 + 3x_2 - x_3 = 0 \\ x_1 - x_2 + 3x_3 = 7 \end{cases}$$

$$4.10 \begin{cases} 2x_1 - 4x_2 + 9x_3 = 28 \\ 7x_1 + 3x_2 - 6x_3 = -1 \\ 7x_1 + 9x_2 - 9x_3 = 5 \end{cases}$$

$$4.11 \begin{cases} x_1 + x_2 + x_3 = 3 \\ 2x_1 + x_2 - 2x_3 = 1 \\ x_1 + x_2 - 3x_3 = -1 \end{cases}$$

$$4.12 \begin{cases} 2x_1 + 2x_2 + 2x_3 = 6 \\ x_1 + 2x_2 + 3x_3 = 1 \\ x_1 + 3x_2 + 6x_3 = 2 \end{cases}$$

$$4.13 \begin{cases} 2x_1 - 4x_2 + x_3 = 3 \\ x_1 - 5x_2 + 3x_3 = -1 \\ x_1 - x_2 + x_3 = 1 \end{cases}$$

$$4.14 \begin{cases} x_1 - x_2 + x_3 = 6 \\ 2x_1 + x_2 + x_3 = 3 \\ x_1 + x_2 + 2x_3 = 5 \end{cases}$$

$$4.15 \begin{cases} 2x_1 + x_2 + x_3 = 6 \\ x_1 - x_2 + x_3 = 5 \\ x_1 + x_2 + 2x_3 = 4 \end{cases}$$

$$4.16 \begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 2 \\ x_1 + 3x_2 + 6x_3 = 1 \end{cases}$$

$$4.17 \begin{cases} x_1 - 2x_3 = 4 \\ 2x_2 + x_3 = 3 \\ x_1 - x_3 = 6 \end{cases}$$

$$4.18 \begin{cases} 2x_1 - x_2 + x_3 = 4 \\ -x_1 + 3x_2 - 2x_3 = -3 \\ 3x_1 + 2x_2 - x_3 = 3 \end{cases}$$

$$4.19 \begin{cases} 3x_1 + x_2 - 2x_3 = -2 \\ x_1 + x_2 + x_3 = 0 \\ x_1 - 2x_2 + 3x_3 = -3 \end{cases}$$

$$4.20 \begin{cases} x_1 + 2x_2 - x_3 = 1 \\ -2x_1 - 3x_2 + 2x_3 = 0 \\ x_1 + 5x_2 + x_3 = -5 \end{cases}$$

$$4.21 \begin{cases} 2x_1 - 3x_2 + x_3 = -3 \\ x_1 - 5x_2 - 2x_3 = 6 \\ 2x_1 + x_2 - 3x_3 = 9 \end{cases}$$

$$4.22 \begin{cases} 3x_1 + x_2 + x_3 = 4 \\ x_1 + 2x_2 + 2x_3 = 3 \\ x_1 + 4x_2 - x_3 = -2 \end{cases}$$

$$4.23 \begin{cases} 4x_1 - x_2 - x_3 = -3 \\ x_1 + 3x_2 + 3x_3 = -4 \\ -x_1 + 2x_2 - x_3 = 5 \end{cases}$$

$$4.24 \begin{cases} 2x_1 - x_2 = 5 \\ 5x_1 + 3x_2 - 6x_3 = -5 \\ -x_1 - 2x_2 + 3x_3 = 7 \end{cases}$$

$$4.25 \begin{cases} 3x_1 - x_2 + x_3 = 12 \\ x_1 + 2x_2 - x_3 = 12 \\ 2x_1 - x_2 + 3x_3 = 9 \end{cases}$$

$$4.26 \begin{cases} x_1 + 3x_2 - 2x_3 = -4 \\ x_1 - x_2 + 4x_3 = 4 \\ 3x_1 + 2x_2 - x_3 = -9 \end{cases}$$

$$4.27 \begin{cases} 2x_1 + x_2 + x_3 = 6 \\ 2x_2 + x_3 = 13 \\ 3x_1 + x_2 + 2x_3 = 8 \end{cases}$$

$$4.28 \begin{cases} 2x_1 - x_2 + 3x_3 = 8 \\ x_1 + x_2 - 2x_3 = 5 \\ 3x_1 - 2x_2 + x_3 = 7 \end{cases}$$

$$4.29 \begin{cases} x_1 + 3x_2 - x_3 = 4 \\ -x_1 + 2x_2 + 3x_3 = 12 \\ 2x_1 + x_2 - x_3 = 1 \end{cases}$$

$$4.30 \begin{cases} x_1 - 3x_2 + 2x_3 = -11 \\ 5x_1 - x_2 + x_3 = -17 \\ 2x_1 + x_2 + x_3 = 0 \end{cases}$$

5. Найти общее решение СЛАУ методом Гаусса.

$$5.1 \begin{cases} x_1 + 2x_2 - 2x_3 - 3x_4 = 4 \\ 2x_1 + 5x_2 - x_3 - 4x_4 = 9 \\ x_1 + 3x_2 + x_3 - x_4 = 5 \end{cases}$$

$$5.3 \begin{cases} x_1 + 2x_2 - 3x_3 - 4x_4 = 1 \\ 3x_1 + 7x_2 - 2x_3 - 4x_4 = 4 \\ 2x_1 + 5x_2 + x_3 + 3x_4 = 3 \end{cases}$$

$$5.5 \begin{cases} x_1 + 3x_2 - x_3 - 2x_4 = 1 \\ 2x_1 + 7x_2 - 4x_3 - 3x_4 = 3 \\ x_1 + 4x_2 - 3x_3 - x_4 = 2 \end{cases}$$

$$5.7 \begin{cases} x_1 - 2x_2 + 2x_3 + 3x_4 = 0 \\ 2x_1 - 3x_2 + x_3 + 4x_4 = 1 \\ 3x_1 - 5x_2 + 3x_3 + 7x_4 = 1 \end{cases}$$

$$5.9 \begin{cases} x_1 - 2x_2 + 2x_3 + 3x_4 = 0 \\ 3x_1 - 5x_2 + x_3 + 4x_4 = 1 \\ 2x_1 - 3x_2 - x_3 + x_4 = 1 \end{cases}$$

$$5.11 \begin{cases} x_1 - 3x_2 + x_3 + 2x_4 = 4 \\ 2x_1 - 5x_2 + 4x_3 + 3x_4 = 7 \\ x_1 - 2x_2 + 3x_3 + x_4 = 3 \end{cases}$$

$$5.13 \begin{cases} x_1 + 4x_2 - 2x_3 - 3x_4 = 2 \\ 2x_1 + 9x_2 - x_3 - 4x_4 = 5 \\ x_1 + 5x_2 + x_3 - x_4 = 3 \end{cases}$$

$$5.15 \begin{cases} x_1 - x_2 + 3x_3 + 4x_4 = 0 \\ 2x_1 - x_2 + 2x_3 + x_4 = 1 \\ 4x_1 - 3x_2 + 8x_3 + 9x_4 = 1 \end{cases}$$

$$5.17 \begin{cases} x_1 - 4x_2 + 2x_3 + 3x_4 = 5 \\ 2x_1 - 7x_2 + 4x_3 + x_4 = 9 \\ x_1 - 3x_2 + 2x_3 - 2x_4 = 4 \end{cases}$$

$$5.19 \begin{cases} x_1 - 5x_2 + 3x_3 + 4x_4 = 4 \\ 2x_1 - 9x_2 + 2x_3 + x_4 = 7 \\ x_1 - 4x_2 - x_3 - 3x_4 = 3 \end{cases}$$

$$5.21 \begin{cases} x_1 + x_2 + 4x_3 + 2x_4 = 0 \\ 3x_1 + 4x_2 + x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - 3x_3 + x_4 = 1 \end{cases}$$

$$5.2 \begin{cases} x_1 - 4x_2 + 2x_3 + 3x_5 = 5 \\ 2x_1 - 7x_2 + 4x_3 + x_4 = 9 \\ x_1 - 3x_2 + 2x_3 + x_4 - 3x_5 = 4 \end{cases}$$

$$5.4 \begin{cases} x_1 - 5x_2 + 3x_3 + 4x_4 = 4 \\ 2x_1 - 9x_2 + 2x_3 + x_5 = 7 \\ x_1 - 4x_2 - x_3 - 4x_4 + x_5 = 3 \end{cases}$$

$$5.6 \begin{cases} x_1 + x_2 + 4x_3 + 2x_5 = 0 \\ 3x_1 + 4x_2 + x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - 3x_3 + 3x_4 - 2x_5 = 1 \end{cases}$$

$$5.8 \begin{cases} x_1 - x_2 + 4x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 + x_3 + 2x_5 = 1 \\ 2x_1 - x_2 - 3x_3 - 3x_4 + 2x_5 = 1 \end{cases}$$

$$5.10 \begin{cases} x_1 - 3x_2 + 4x_3 + 3x_5 = 2 \\ 3x_1 - 8x_2 + x_3 + 2x_4 = 5 \\ 2x_1 - 5x_2 - 3x_3 + 2x_4 - 3x_5 = 3 \end{cases}$$

$$5.12 \begin{cases} x_1 - x_2 + 3x_3 + 4x_4 = 0 \\ 4x_1 - 3x_2 + x_3 + 2x_5 = 1 \\ 3x_1 - 2x_2 - 2x_3 - 4x_4 + 2x_5 = 1 \end{cases}$$

$$5.14 \begin{cases} x_1 - 2x_2 + 3x_3 + 4x_5 = 1 \\ 4x_1 - 7x_2 + 2x_3 + x_4 = 3 \\ 3x_1 - 5x_2 - x_3 + x_4 - 4x_5 = 2 \end{cases}$$

$$5.16 \begin{cases} x_1 + x_2 - 3x_3 - 4x_4 = 1 \\ 4x_1 + 5x_2 - 2x_3 - x_5 = 3 \\ 3x_1 + 4x_2 + x_3 + 4x_4 - x_5 = 2 \end{cases}$$

$$5.18 \begin{cases} x_1 + 2x_2 - 2x_3 - 3x_5 = 4 \\ 2x_1 + 5x_2 - x_3 - 4x_4 = 9 \\ x_1 + 3x_2 + x_3 - 4x_4 + 3x_5 = 5 \end{cases}$$

$$5.20 \begin{cases} x_1 + 2x_2 - 3x_3 - 4x_4 = 1 \\ 3x_1 + 7x_2 - 2x_3 + x_5 = 4 \\ 2x_1 + 5x_2 + x_3 + 4x_4 + x_5 = 3 \end{cases}$$

$$5.22 \begin{cases} x_1 + 3x_2 - x_3 - 2x_5 = 1 \\ 2x_1 + 7x_2 - 4x_3 - 3x_4 = 3 \\ x_1 + 4x_2 - 3x_3 - 3x_4 + 2x_5 = 2 \end{cases}$$

$$5.23 \begin{cases} x_1 - x_2 + 4x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 + x_3 + 2x_4 = 1 \\ 2x_1 - x_2 - 3x_3 - x_4 = 1 \end{cases}$$

$$5.25 \begin{cases} x_1 - 3x_2 + 4x_3 + 3x_4 = 2 \\ 3x_1 - 8x_2 + x_3 + 2x_4 = 5 \\ 2x_1 - 5x_2 - 3x_3 - x_4 = 3 \end{cases}$$

$$5.27 \begin{cases} x_1 - x_2 + 3x_3 + 4x_4 = 0 \\ 4x_1 - 3x_2 + x_3 + 2x_4 = 1 \\ 3x_1 - 2x_2 - 2x_3 - 2x_4 = 1 \end{cases}$$

$$5.29 \begin{cases} x_1 - 2x_2 + 3x_3 + 4x_4 = 1 \\ 4x_1 - 7x_2 + 2x_3 + x_4 = 3 \\ 3x_1 - 5x_2 - x_3 - 3x_4 = 2 \end{cases}$$

$$5.24 \begin{cases} x_1 - 2x_2 + 2x_3 + 3x_4 = 0 \\ 2x_1 - 3x_2 + x_3 + 4x_5 = 1 \\ 3x_1 - 5x_2 + 3x_3 + 3x_4 + 4x_5 = 1 \end{cases}$$

$$5.26 \begin{cases} x_1 - 2x_2 + 2x_3 + 3x_5 = 0 \\ 3x_1 - 5x_2 + x_3 + 4x_4 = 1 \\ 2x_1 - 3x_2 - x_3 + 4x_4 - 3x_5 = 1 \end{cases}$$

$$5.28 \begin{cases} x_1 - 3x_2 + x_3 + 2x_4 = 4 \\ 2x_1 - 5x_2 + 4x_3 + 3x_5 = 7 \\ x_1 - 2x_2 + 3x_3 - 2x_4 + 3x_5 = 3 \end{cases}$$

$$5.30 \begin{cases} x_1 + 4x_2 - 2x_3 - 3x_5 = 2 \\ 2x_1 + 9x_2 - x_3 - 4x_4 = 5 \\ x_1 + 5x_2 + x_3 - 4x_4 + 3x_5 = 3 \end{cases}$$

6. Решить матричное уравнение.

$$6.1. \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 11 \\ 2 & 6 \end{pmatrix}.$$

$$6.3. \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \cdot X = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}.$$

$$6.5. \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}.$$

$$6.7. \begin{pmatrix} -3 & 1 \\ 2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} -8 & -5 \\ 5 & 3 \end{pmatrix}.$$

$$6.9. \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}.$$

$$6.11. \begin{pmatrix} 2 & 3 \\ 1 & -3 \end{pmatrix} \cdot X = \begin{pmatrix} 0 & 9 \\ 3 & 0 \end{pmatrix}.$$

$$6.13. \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} -3 & 2 \\ 3 & -1 \end{pmatrix}.$$

$$6.15. \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 8 & 6 \\ -1 & -2 \end{pmatrix}.$$

$$6.17. \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 5 \\ 6 & 9 \end{pmatrix}.$$

$$6.19. \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 7 & -4 \\ -6 & 2 \end{pmatrix}.$$

$$6.2. X \cdot \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -1 & -2 \end{pmatrix}.$$

$$6.4. X \cdot \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 6 & 9 \end{pmatrix}.$$

$$6.6. X \cdot \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & -4 \\ -6 & 2 \end{pmatrix}.$$

$$6.8. X \cdot \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -9 \\ -2 & 5 \end{pmatrix}.$$

$$6.10. X \cdot \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 4 & 7 \end{pmatrix}.$$

$$6.12. X \cdot \begin{pmatrix} -2 & 4 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} -3 & 7 \\ 6 & -10 \end{pmatrix}.$$

$$6.14. X \cdot \begin{pmatrix} 5 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ -2 & 1 \end{pmatrix}.$$

$$6.16. X \cdot \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 2 & 6 \end{pmatrix}.$$

$$6.18. X \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}.$$

$$6.20. X \cdot \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}.$$

$$6.21. \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & -9 \\ -2 & 5 \end{pmatrix}.$$

$$6.22. X \cdot \begin{pmatrix} -3 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -8 & -5 \\ 5 & 3 \end{pmatrix}.$$

$$6.23. \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} -1 & -8 \\ 4 & 7 \end{pmatrix}.$$

$$6.24. X \cdot \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}.$$

$$6.25. \begin{pmatrix} 5 & 1 \\ 3 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 7 & 1 \\ -2 & 0 \end{pmatrix}.$$

$$6.26. X \cdot \begin{pmatrix} 2 & 3 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 3 & 0 \end{pmatrix}.$$

$$6.27. \begin{pmatrix} -2 & 4 \\ -3 & 5 \end{pmatrix} \cdot X = \begin{pmatrix} -3 & 7 \\ 6 & -10 \end{pmatrix}.$$

$$6.28. X \cdot \begin{pmatrix} 5 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ -2 & 0 \end{pmatrix}.$$

$$6.29. \begin{pmatrix} 5 & -1 \\ -1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 6 & 3 \\ -2 & 1 \end{pmatrix}.$$

$$6.30. X \cdot \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 3 & -1 \end{pmatrix}.$$

7. Даны векторы $\bar{a}, \bar{b}, \bar{c}, \bar{d}$. Требуется вычислить скалярное произведение векторов $\bar{m} \cdot \bar{n}$, если $\bar{m} = \bar{a} + 2\bar{b}$, $\bar{n} = 3\bar{c} - \bar{d}$ и установить ортогональность векторов $\bar{m}$ и $\bar{n}$.

$$7.1. \bar{a} = (1, -1, -1), \quad \bar{b} = (0, 1, 1), \quad \bar{c} = (-1, 0, 1), \quad \bar{d} = (-3, 3, 5).$$

$$7.2. \bar{a} = (1, 1, 0), \quad \bar{b} = (-1, 0, -1), \quad \bar{c} = (1, 1, 1), \quad \bar{d} = (0, 2, -1).$$

$$7.3. \bar{a} = (2, -3, 1), \quad \bar{b} = (3, -3, 1), \quad \bar{c} = (2, -1, 2), \quad \bar{d} = (6, -8, 1).$$

$$7.4. \bar{a} = (1, 1, 0), \quad \bar{b} = (1, 0, 1), \quad \bar{c} = (1, 1, 1), \quad \bar{d} = (2, 3, 1)$$

$$7.5. \bar{a} = (2, 1, 3), \quad \bar{b} = (4, 2, 1), \quad \bar{c} = (3, 4, 5), \quad \bar{d} = (1, 3, 2).$$

$$7.6. \bar{a} = (2, 3, 1), \quad \bar{b} = (-1, 2, -2), \quad \bar{c} = (1, 2, 1), \quad \bar{d} = (2, -2, 1).$$

$$7.7. \bar{a} = (1, 2, 1), \quad \bar{b} = (2, -1, 3), \quad \bar{c} = (3, -1, 4), \quad \bar{d} = (5, 1, 6).$$

$$7.8. \bar{a} = (-1, 0, 2), \quad \bar{b} = (-2, 2, 1), \quad \bar{c} = (1, -3, 2), \quad \bar{d} = (1, 4, -6).$$

$$7.9. \bar{a} = (1, -1, 0), \quad \bar{b} = (0, 1, -1), \quad \bar{c} = (0, 0, 1), \quad \bar{d} = (2, -1, 0).$$

$$7.10. \bar{a} = (3, 3, -1), \quad \bar{b} = (3, 1, 0), \quad \bar{c} = (-1, 2, 1), \quad \bar{d} = (-1, 0, 2).$$

$$7.11. \bar{a} = (0, 1, 2), \quad \bar{b} = (1, 0, 1), \quad \bar{c} = (-1, 2, 4), \quad \bar{d} = (-2, 4, 7).$$

$$7.12. \bar{a} = (1, 3, 0), \quad \bar{b} = (2, -1, 1), \quad \bar{c} = (0, -1, 2), \quad \bar{d} = (6, 12, -1).$$

$$7.13. \bar{a} = (2, 1, -1), \quad \bar{b} = (0, 3, 2), \quad \bar{c} = (1, -1, 1), \quad \bar{d} = (1, -4, 4).$$

$$7.14. \bar{a} = (4, 1, 1), \quad \bar{b} = (2, 0, -3), \quad \bar{c} = (-1, 2, 1), \quad \bar{d} = (-9, 5, 5).$$

$$7.15. \bar{a} = (-2, 0, 1), \quad \bar{b} = (1, 3, -1), \quad \bar{c} = (0, 4, 1), \quad \bar{d} = (-5, -5, 5).$$

$$7.16. \bar{a} = (5, 1, 0), \quad \bar{b} = (2, -1, 3), \quad \bar{c} = (1, 0, -1), \quad \bar{d} = (13, 2, 7).$$

$$7.17. \bar{a} = (0, 1, 1), \quad \bar{b} = (-2, 0, 1), \quad \bar{c} = (3, 1, 0), \quad \bar{d} = (-19, -1, 7).$$

$$7.18. \bar{a} = (1, 0, 2), \quad \bar{b} = (0, 1, 1), \quad \bar{c} = (2, -1, 4), \quad \bar{d} = (3, -3, 4).$$

$$7.19. \bar{a} = (3, 1, 0), \quad \bar{b} = (-1, 2, 1), \quad \bar{c} = (-1, 0, 2), \quad \bar{d} = (3, 3, -1).$$

$$7.20. \bar{a} = (-1, 2, 1), \quad \bar{b} = (2, 0, 3), \quad \bar{c} = (1, 1, -1), \quad \bar{d} = (-1, 7, -4).$$

$$7.21. \bar{a} = (1, 1, 4), \quad \bar{b} = (0, -3, 2), \quad \bar{c} = (2, 1, -1), \quad \bar{d} = (6, 5, -14).$$

$$7.22. \bar{a} = (1, -2, 0), \quad \bar{b} = (-1, 1, 3), \quad \bar{c} = (1, 0, 4), \quad \bar{d} = (6, -1, 7).$$

$$7.23. \bar{a} = (2, 0, 1), \quad \bar{b} = (1, 1, 0), \quad \bar{c} = (4, 1, 2), \quad \bar{d} = (8, 0, 5).$$

$$7.24. \bar{a} = (0, 1, 3), \quad \bar{b} = (1, 2, -1), \quad \bar{c} = (2, 0, -1), \quad \bar{d} = (3, 1, 8).$$

$$7.25. \bar{a} = (1, 0, 5), \quad \bar{b} = (-1, 3, 2), \quad \bar{c} = (0, -1, 1), \quad \bar{d} = (5, 15, 0).$$

$$7.26. \bar{a} = (1, 1, 0), \quad \bar{b} = (0, 1, -2), \quad \bar{c} = (1, 0, 3), \quad \bar{d} = (2, -1, 11).$$

$$7.27. \bar{a} = (1, 0, 2), \quad \bar{b} = (-1, 0, 1), \quad \bar{c} = (2, 5, -3), \quad \bar{d} = (11, 5, -3).$$

$$7.28. \bar{a} = (1, 2, -1), \quad \bar{b} = (3, 0, 2), \quad \bar{c} = (-1, 1, 1), \quad \bar{d} = (8, 1, 12).$$

$$7.29. \bar{a} = (1, 0, 1), \quad \bar{b} = (0, -2, 1), \quad \bar{c} = (1, 3, 0), \quad \bar{d} = (8, 9, 4).$$

$$7.30. \bar{a} = (2, 1, 0), \quad \bar{b} = (1, 0, 1), \quad \bar{c} = (4, 2, 1), \quad \bar{d} = (3, 1, 3).$$

8. Даны точки A, B . Требуется найти координаты, модуль и направляющие косинусы вектора $\overrightarrow{AB}$.

- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| 8.1. $A(1, 1, 3), B(2, 2, 3)$ | 8.2. $A(0, 1, 3), B(1, 2, 3)$ | 8.3. $A(0, 1, -1), B(1, 2, 0)$ |
| 8.4. $A(2, 2, 3), B(3, 2, 4)$ | 8.5. $A(2, 1, 2), B(3, 2, 2)$ | 8.6. $A(0, 1, 1), B(1, 2, 2)$ |
| 8.7. $A(0, 1, 4), B(1, 2, 4)$ | 8.8. $A(1, 1, 1), B(1, 2, 2)$ | 8.9. $A(0, -4, 3), B(1, -3, 4)$ |
| 8.10. $A(1, 2, 1), B(0, 1, 2)$ | 8.11. $A(2, 1, 3), B(3, 2, 4)$ | 8.12. $A(0, 1, 1), B(2, 1, 1)$ |
| 8.13. $A(2, 1, 3), B(4, 2, 3)$ | 8.14. $A(2, 0, 7), B(0, 2, 4)$ | 8.15. $A(8, 2, -5), B(7, 1, 4)$ |
| 8.16. $A(-2, 1, 3), B(5, 1, 2)$ | 8.17. $A(2, -1, 4), B(5, 2, 3)$ | 8.18. $A(3, 1, 3), B(2, 2, -1)$ |
| 8.19. $A(2, 2, 3), B(3, 1, 3)$ | 8.20. $A(0, 7, 3), B(4, 7, -5)$ | 8.21. $A(4, -3, 2), B(1, 2, 3)$ |
| 8.22. $A(5, 1, 1), B(6, 2, 1)$ | 8.23. $A(0, 4, 2), B(3, 6, -4)$ | 8.24. $A(1, 3, 2), B(4, 6, 5)$ |
| 8.25. $A(0, -2, 1), B(2, 0, 3)$ | 8.26. $A(2, -2, 3), B(2, 1, 7)$ | 8.27. $A(1, 3, 3), B(2, 4, 2)$ |
| 8.28. $A(2, 0, -1), B(4, 2, 0)$ | 8.29. $A(1, 3, -2), B(3, 2, 0)$ | 8.30. $A(1, 3, -1), B(3, 1, 0)$ |

9. Даны вершины треугольника ABC . Требуется найти:

а) длину стороны AB ;

б) косинус угла между сторонами AB и AC .

- | | |
|--------------------------------------|--------------------------------------|
| 9.1 $A(2, -5), B(-3, -4), C(-4, 2)$ | 9.2 $A(4, -5), B(2, 2), C(7, 4)$ |
| 9.3 $A(3, 2), B(-5, 4), C(-1, -6)$ | 9.4 $A(-3, -4), B(-6, 7), C(-1, 1)$ |
| 9.5 $A(-3, -5), B(4, 3), C(-2, 4)$ | 9.6 $A(2, 1), B(-7, 3), C(-4, -3)$ |
| 9.7 $A(-4, 5), B(3, -3), C(5, 2)$ | 9.8 $A(3, -4), B(2, 1), C(1, 7)$ |
| 9.9 $A(-4, -5), B(-2, 2), C(-7, 4)$ | 9.10 $A(3, 4), B(6, 7), C(1, 1)$ |
| 9.11 $A(-2, -1), B(7, 3), C(4, -3)$ | 9.12 $A(6, -4), B(-3, -7), C(-1, 2)$ |
| 9.13 $A(3, 2), B(2, -5), C(-6, -1)$ | 9.14 $A(-6, -4), B(3, -7), C(1, 2)$ |
| 9.15 $A(-3, 2), B(-2, -5), C(6, -1)$ | 9.16 $A(2, 5), B(-3, 4), C(-4, -2)$ |
| 9.17 $A(3, -2), B(-5, -4), C(-1, 6)$ | 9.18 $A(-3, 5), B(4, -3), C(-2, -4)$ |
| 9.19 $A(-4, -5), B(3, 3), C(5, -2)$ | 9.20 $A(3, 4), B(2, -1), C(1, -7)$ |
| 9.21 $A(-6, 5), B(6, 0), C(9, 4)$ | 9.22 $A(3, -3), B(-1, -6), C(-6, 6)$ |

9.23 $A(-8, 4), B(4, -1), C(7, 3)$

9.25 $A(7, -4), B(3, -7), C(-2, 5)$

9.27 $A(6, 0), B(2, -3), C(-3, 9)$

9.29 $A(-7, 3), B(5, -2), C(8, 2)$

9.24 $A(-14, 6), B(-2, 1), C(1, 5)$

9.26 $A(-9, 2), B(3, -3), C(6, 1)$

9.28 $A(5, -1), B(1, -4), C(-4, 8)$

9.30 $A(4, 1), B(0, -2), C(-5, 10)$

10. Даны вершины треугольника ABC . Требуется найти:

а) уравнение стороны AB ;

б) расстояние от вершины C до стороны AB ;

в) уравнение медианы BE , проведённой из вершины B ;

г) уравнение высоты CD , проведённой из вершины C .

10.1 $A(4, 1), B(0, -2), C(-5, 10)$.

10.3 $A(5, -1), B(1, -4), C(-4, 8)$

10.5 $A(-9, 2), B(3, -3), C(6, 1)$

10.7 $A(-14, 6), B(-2, 1), C(1, 5)$

10.9 $A(3, -3), B(-1, -6), C(-6, 6)$

10.11 $A(3, 4), B(2, -1), C(1, -7)$

10.13 $A(-3, 5), B(4, -3), C(-2, -4)$

10.15 $A(2, 5), B(-3, 4), C(-4, -2)$

10.17 $A(-6, -4), B(3, -7), C(1, 2)$

10.19 $A(6, -4), B(-3, -7), C(-1, 2)$

10.21 $A(3, 4), B(6, 7), C(1, 1)$

10.23 $A(3, -4), B(2, 1), C(1, 7)$

10.25 $A(2, 1), B(-7, 3), C(-4, -3)$

10.27 $A(-3, -4), B(-6, 7), C(-1, 1)$

10.29 $A(4, -5), B(2, 2), C(7, 4)$

10.2 $A(-7, 3), B(5, -2), C(8, 2)$.

10.4 $A(6, 0), B(2, -3), C(-3, 9)$

10.6 $A(7, -4), B(3, -7), C(-2, 5)$

10.8 $A(-8, 4), B(4, -1), C(7, 3)$

10.10 $A(-6, 5), B(6, 0), C(9, 4)$

10.12 $A(-4, -5), B(3, 3), C(5, -2)$

10.14 $A(3, -2), B(-5, -4), C(-1, 6)$

10.16 $A(-3, 2), B(-2, -5), C(6, -1)$

10.18 $A(3, 2), B(2, -5), C(-6, -1)$

10.20 $A(-2, -1), B(7, 3), C(4, -3)$

10.22 $A(-4, -5), B(-2, 2), C(-7, 4)$

10.24 $A(-4, 5), B(3, -3), C(5, 2)$

10.26 $A(-3, -5), B(4, 3), C(-2, 4)$

10.28 $A(3, 2), B(-5, 4), C(-1, -6)$

10.30 $A(2, -5), B(-3, -4), C(-4, 2)$

11. Построить кривые по заданным уравнениям и назвать их.

11.1 $(x-2)^2 + (y-3)^2 = 9; \frac{x^2}{25} + \frac{y^2}{9} = 1; \frac{x^2}{49} - \frac{y^2}{25} = 1; y^2 = 9x.$

11.2 $(x-3)^2 + (y-2)^2 = 9; \frac{x^2}{9} + \frac{y^2}{25} = 1; \frac{x^2}{49} - \frac{y^2}{25} = -1; y^2 = -4x.$

11.3 $(x+2)^2 + (y-1)^2 = 36; \frac{x^2}{9} + \frac{y^2}{4} = 1; \frac{x^2}{16} - \frac{y^2}{4} = 1; y^2 = 2x.$

11.4 $(x-1)^2 + (y+2)^2 = 36; \frac{x^2}{4} + \frac{y^2}{9} = 1; \frac{x^2}{16} - \frac{y^2}{4} = -1; x^2 = 7y.$

11.5 $(x+3)^2 + (y-5)^2 = 4; \frac{x^2}{49} + \frac{y^2}{4} = 1; \frac{x^2}{25} - \frac{y^2}{16} = 1; y^2 = 7x.$

11.6 $(x-5)^2 + (y+3)^2 = 4; \frac{x^2}{4} + \frac{y^2}{49} = 1; \frac{x^2}{25} - \frac{y^2}{16} = -1; y^2 = -2x.$

- 11.7 $(x-4)^2 + (y+2)^2 = 49$; $\frac{x^2}{49} + \frac{y^2}{36} = 1$; $\frac{x^2}{25} - \frac{y^2}{9} = 1$; $y^2 = 6x$.
- 11.8 $(x+2)^2 + (y-4)^2 = 49$; $\frac{x^2}{36} + \frac{y^2}{49} = 1$; $\frac{x^2}{25} - \frac{y^2}{9} = -1$; $x^2 = 5y$.
- 11.9 $(x+1)^2 + (y-2)^2 = 16$; $\frac{x^2}{36} + \frac{y^2}{25} = 1$; $\frac{x^2}{16} - \frac{y^2}{9} = 1$; $y^2 = 5x$.
- 11.10 $(x+1)^2 + (y+1)^2 = 16$; $\frac{x^2}{25} + \frac{y^2}{36} = 1$; $\frac{x^2}{16} - \frac{y^2}{9} = -1$; $y^2 = -6x$.
- 11.11 $(x+4)^2 + (y-4)^2 = 9$; $\frac{x^2}{36} + \frac{y^2}{9} = 1$; $\frac{x^2}{36} - \frac{y^2}{16} = 1$; $y^2 = x$.
- 11.12 $(x-4)^2 + (y+4)^2 = 9$; $\frac{x^2}{9} + \frac{y^2}{36} = 1$; $\frac{x^2}{36} - \frac{y^2}{16} = -1$; $x^2 = 16y$.
- 11.13 $(x-3)^2 + (y+4)^2 = 25$; $\frac{x^2}{25} + \frac{y^2}{16} = 1$; $\frac{x^2}{64} - \frac{y^2}{25} = 1$; $y^2 = 16x$.
- 11.14 $(x+4)^2 + (y-3)^2 = 25$; $\frac{x^2}{16} + \frac{y^2}{25} = 1$; $\frac{x^2}{64} - \frac{y^2}{25} = -1$; $y^2 = -x$.
- 11.15 $(x-5)^2 + (y+1)^2 = 4$; $\frac{x^2}{16} + \frac{y^2}{9} = 1$; $\frac{x^2}{49} - \frac{y^2}{9} = 1$; $y^2 = 8x$.
- 11.16 $(x+1)^2 + (y-5)^2 = 4$; $\frac{x^2}{9} + \frac{y^2}{16} = 1$; $\frac{x^2}{49} - \frac{y^2}{9} = -1$; $x^2 = 3y$.
- 11.17 $(x+3)^2 + (y+3)^2 = 4$; $\frac{x^2}{49} + \frac{y^2}{25} = 1$; $\frac{x^2}{36} - \frac{y^2}{9} = 1$; $y^2 = 3x$.
- 11.18 $(x-3)^2 + (y-3)^2 = 4$; $\frac{x^2}{25} + \frac{y^2}{49} = 1$; $\frac{x^2}{36} - \frac{y^2}{9} = -1$; $y^2 = -8x$.
- 11.19 $(x+5)^2 + (y-6)^2 = 16$; $\frac{x^2}{25} + \frac{y^2}{4} = 1$; $\frac{x^2}{36} - \frac{y^2}{25} = 1$; $y^2 = -9x$.
- 11.20 $(x-6)^2 + (y+5)^2 = 16$; $\frac{x^2}{4} + \frac{y^2}{25} = 1$; $\frac{x^2}{36} - \frac{y^2}{25} = -1$; $x^2 = 4y$.
- 11.21 $(x-1)^2 + (y+1)^2 = 1$; $\frac{x^2}{16} + \frac{y^2}{4} = 1$; $\frac{x^2}{9} - \frac{y^2}{4} = 1$; $y^2 = 4x$.
- 11.22 $(x+1)^2 + (y-1)^2 = 1$; $\frac{x^2}{4} + \frac{y^2}{16} = 1$; $\frac{x^2}{9} - \frac{y^2}{4} = -1$; $x^2 = 9y$.
- 11.23 $(x-1)^2 + (y+5)^2 = 1$; $\frac{x^2}{36} + \frac{y^2}{16} = 1$; $\frac{x^2}{25} - \frac{y^2}{4} = 1$; $y^2 = -7x$.
- 11.24 $(x+5)^2 + (y-1)^2 = 1$; $\frac{x^2}{16} + \frac{y^2}{36} = 1$; $\frac{x^2}{25} - \frac{y^2}{4} = -1$; $x^2 = 2y$.
- 11.25 $(x+1)^2 + (y-3)^2 = 25$; $\frac{x^2}{49} + \frac{y^2}{9} = 1$; $\frac{x^2}{49} - \frac{y^2}{16} = 1$; $y^2 = -5x$.

$$11.26 \quad (x-3)^2 + (y+1)^2 = 25; \frac{x^2}{9} + \frac{y^2}{49} = 1; \frac{x^2}{49} - \frac{y^2}{16} = -1; x^2 = 6y.$$

$$11.27 \quad (x-3)^2 + (y-2)^2 = 36; \frac{x^2}{64} + \frac{y^2}{25} = 1; \frac{x^2}{36} - \frac{y^2}{4} = 1; y^2 = -16x.$$

$$11.28 \quad (x-2)^2 + (y-3)^2 = 36; \frac{x^2}{16} + \frac{y^2}{49} = 1; \frac{x^2}{36} - \frac{y^2}{4} = -1; x^2 = y.$$

$$11.29 \quad (x-2)^2 + (y-3)^2 = 9; \frac{x^2}{25} + \frac{y^2}{9} = 1; \frac{x^2}{49} - \frac{y^2}{25} = 1; y^2 = 9x.$$

$$11.30 \quad (x-3)^2 + (y-2)^2 = 9; \frac{x^2}{9} + \frac{y^2}{25} = 1; \frac{x^2}{49} - \frac{y^2}{25} = -1; y^2 = -4x.$$

12. Уравнение определяет невырожденную действительную кривую второго порядка. Требуется: назвать кривую, привести её уравнение к нормальному виду и построить.

$$12.1 \quad 4x^2 + y^2 - 8x + 4y = 0$$

$$12.3 \quad 2x^2 + 3y^2 + 12x - 6y + 9 = 0$$

$$12.5 \quad 9x^2 + 16y^2 + 36x - 64y - 44 = 0$$

$$12.7 \quad 9x^2 + 4y^2 + 36x - 8y + 36 = 0$$

$$12.9 \quad 4x^2 + 25y^2 - 8x + 100y + 4 = 0$$

$$12.11 \quad x^2 + 6x - 2y + 5 = 0$$

$$12.13 \quad 4x^2 + 9y^2 - 32x + 36y + 64 = 0$$

$$12.15 \quad 9x^2 + 4y^2 + 18x - 8y + 9 = 0$$

$$12.17 \quad x^2 + 25y^2 + 4x - 150y + 204 = 0$$

$$12.19 \quad 3x^2 + 2y^2 + 12x - 16y + 40 = 0$$

$$12.21 \quad x^2 - 8x + 2y + 18 = 0$$

$$12.23 \quad 4x^2 + y^2 - 8x + 4y + 4 = 0$$

$$12.25 \quad x^2 + 4y^2 + 10x - 24y + 57 = 0$$

$$12.27 \quad 4x^2 + 9y^2 + 32x - 18y + 9 = 0$$

$$12.29 \quad 4x^2 + y^2 - 8x + 4y = 0$$

$$12.2 \quad 9x^2 - 4y^2 + 54x + 8y + 41 = 0$$

$$12.4 \quad 4x^2 - y^2 + 8x - 2y + 7 = 0$$

$$12.6 \quad y^2 - 4x - 8y = 0$$

$$12.8 \quad x^2 - 4y^2 + 10x + 24y - 7 = 0$$

$$12.10 \quad x^2 - 4y^2 + 6x + 8y + 1 = 0$$

$$12.12 \quad 9x^2 - 4y^2 + 36x + 8y + 68 = 0$$

$$12.14 \quad 4x^2 - y^2 - 8x - 4y - 16 = 0$$

$$12.16 \quad x^2 + 4x + 2y + 4 = 0$$

$$12.18 \quad 4x^2 - 9y^2 + 16x + 54y - 101 = 0$$

$$12.20 \quad 9x^2 - 16y^2 - 36x - 64y - 172 = 0$$

$$12.22 \quad 9x^2 - 4y^2 - 18x - 16y + 9 = 0$$

$$12.24 \quad 4x^2 - y^2 - 16x - 6y + 11 = 0$$

$$12.26 \quad 4x^2 + 4x + 3y - 2 = 0$$

$$12.28 \quad 5x^2 + 3y^2 - 10x + 12y + 2 = 0$$

$$12.30 \quad 9x^2 - 4y^2 + 54x + 8y + 41 = 0$$

13. Найти область определения функции

$$13.1 \quad y = \sqrt{9 - x^2}$$

$$13.2 \quad y = \sqrt{x-5} + \sqrt{2-x}$$

$$13.3 \quad y = \frac{1}{\sqrt{x^2 - 4x - 5}}$$

$$13.4 \quad y = \ln\left(\frac{x}{x-3}\right)$$

$$13.5 \quad y = \arcsin(2x-1)$$

$$13.6 \quad y = \sqrt{x^2 - 4}$$

$$13.7 \quad y = \sqrt{x-3} + \sqrt{5-x}$$

$$13.8 \quad y = \sqrt{x^2 - 2x - 3}$$

$$13.9 \quad y = \ln\left(\frac{x-5}{x}\right)$$

13.10 $y = \arccos(2x+3)$	13.11 $y = \sqrt{16-x^2}$	13.12 $y = \sqrt{\frac{x-2}{x+2}}$
13.13 $y = \frac{x}{\sqrt{x^2-3x+2}}$	13.14 $y = 2\ln\left(\frac{x}{x+1}\right)$	13.15 $y = \arcsin\left(\frac{3-2x}{4}\right)$
13.16 $y = \sqrt{x^2+6x-16}$	13.17 $y = \sqrt{x-3} + \sqrt{4-x}$	13.18 $y = \frac{1}{\sqrt{x^2-5x+6}}$
13.19 $y = \ln\left(\frac{x+5}{x-3}\right)$	13.20 $y = \arccos\left(\frac{1-2x}{4}\right)$	13.21 $y = \sqrt{x^2-25}$
13.22 $y = \sqrt{x-4} + \sqrt{6-x}$	13.23 $y = \frac{1}{\sqrt{x^2-9x+14}}$	13.24 $y = \ln\left(\frac{x+6}{x}\right)$
13.25 $y = \arcsin\left(\frac{x-3}{2}\right)$	13.26 $y = \sqrt{\frac{1-x}{\sqrt{1+x}}}$	13.27 $y = \sqrt{x} + \lg(2x-3)$
13.28 $y = \frac{x}{\sqrt{x^2-7x+10}}$	13.29 $y = \ln\left(\frac{x-2}{x+4}\right)$	13.30 $y = \arccos\left(\frac{x+3}{4}\right)$

14. Вычислить пределы.

14.1 а) $\lim_{x \rightarrow \infty} \frac{3x^2+5x+1}{x^2+4}$	б) $\lim_{x \rightarrow -1} \frac{2x^2+5x+3}{x^2-4x-5}$	в) $\lim_{x \rightarrow 7} \frac{x-7}{\sqrt{2x+11}-5}$
14.2 а) $\lim_{x \rightarrow \infty} \frac{2x^3+4}{x^3+2x+1}$	б) $\lim_{x \rightarrow 3} \frac{3x^2-10x+3}{x^2-2x-3}$	в) $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{4x-3}-3}$
14.3 а) $\lim_{x \rightarrow \infty} \frac{x(x+1)(x+2)}{2x^3+5}$	б) $\lim_{x \rightarrow 2} \frac{x^2+6x-16}{3x^2-5x-2}$	в) $\lim_{x \rightarrow 4} \frac{\sqrt{4x}-x}{x^2-16}$
14.4 а) $\lim_{x \rightarrow \infty} \frac{x^2+6x+5}{3x^2+7}$	б) $\lim_{x \rightarrow 1} \frac{x^2-3x+2}{3x^2+4x-7}$	в) $\lim_{x \rightarrow 0} \frac{\sqrt{9-x}-3}{x^2+x}$
14.5 а) $\lim_{x \rightarrow \infty} \frac{x^4+3x+1}{3x^4+5}$	б) $\lim_{x \rightarrow 3} \frac{2x^2-5x-3}{x^2-5x+6}$	в) $\lim_{x \rightarrow 2} \frac{x^2-4}{\sqrt{6x+4}-4}$
14.6 а) $\lim_{x \rightarrow \infty} \frac{x^3-3x+6}{x^2+7x-1}$	б) $\lim_{x \rightarrow 1} \frac{3x^2-4x+1}{2x^2+5x-7}$	в) $\lim_{x \rightarrow 5} \frac{2x-10}{\sqrt{5x}-5}$
14.7 а) $\lim_{x \rightarrow \infty} \frac{3x^2+6x+3}{2x^2+7}$	б) $\lim_{x \rightarrow 7} \frac{2x^2-13x-7}{x^2-9x+14}$	в) $\lim_{x \rightarrow 6} \frac{\sqrt{2x-8}-2}{x-6}$
14.8 а) $\lim_{x \rightarrow \infty} \frac{x^2+4x+5}{3x^2+7x+2}$	б) $\lim_{x \rightarrow 5} \frac{2x^2-11x+5}{x^2-7x+10}$	в) $\lim_{x \rightarrow 5} \frac{3x-15}{\sqrt{2x-1}-3}$
14.9 а) $\lim_{x \rightarrow \infty} \frac{1-2x^2}{3x^2+5x+1}$	б) $\lim_{x \rightarrow 6} \frac{2x^2-9x-18}{x^2-7x+6}$	в) $\lim_{x \rightarrow 6} \frac{\sqrt{2x-8}-2}{x-6}$
14.10 а) $\lim_{x \rightarrow \infty} \frac{4x-5x^3}{2x^2+3x+3}$	б) $\lim_{x \rightarrow 6} \frac{x^2-9x+18}{3x^2-17x-6}$	в) $\lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x+2}-1}$

$$14.11 \text{ a) } \lim_{x \rightarrow \infty} \frac{3x^2 - 5x}{x - 5x^2 - 1}$$

$$14.12 \text{ a) } \lim_{x \rightarrow \infty} \frac{2 + 7x - 2x^2}{x^3 - 5x}$$

$$14.13 \text{ a) } \lim_{x \rightarrow \infty} \frac{4x^2 + x}{3x^2 + 7x - 1}$$

$$14.14 \text{ a) } \lim_{x \rightarrow \infty} \frac{2x^2 - 2x + 5}{3x + x^3}$$

$$14.15 \text{ a) } \lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 2}{x + 3}$$

$$14.16 \text{ a) } \lim_{x \rightarrow \infty} \frac{5x - 3x^2}{x^2 + 4x + 3}$$

$$14.17 \text{ a) } \lim_{x \rightarrow \infty} \frac{5x^2 + 6x - 1}{3x - 2x^2}$$

$$14.18 \text{ a) } \lim_{x \rightarrow \infty} \frac{7x - x^3}{x^2 + 3x + 2}$$

$$14.19 \text{ a) } \lim_{x \rightarrow \infty} \frac{4x^3 + 5x^2}{2x^2 + x^3 + 30}$$

$$14.20 \text{ a) } \lim_{x \rightarrow \infty} \frac{4x + 5x^2}{x^3 - 3x + 1}$$

$$14.21 \text{ a) } \lim_{x \rightarrow \infty} \frac{x^2 + 2x + 6}{3x^2 + 7x - 1}$$

$$14.22 \text{ a) } \lim_{x \rightarrow \infty} \frac{3x^3 + x^2 - 2}{4x^3 + 5x + 1}$$

$$14.23 \text{ a) } \lim_{x \rightarrow \infty} \frac{1 + x - 2x^2}{5x^2 - 8x - 4}$$

$$14.24 \text{ a) } \lim_{x \rightarrow \infty} \frac{x^2 + x + 5}{x^3 - 3x^2 + 2}$$

$$14.25 \text{ a) } \lim_{x \rightarrow \infty} \frac{2 + 5x - 3x^2}{x^2 + 4x + 1}$$

$$14.26 \text{ a) } \lim_{x \rightarrow \infty} \frac{x^4 + 5}{x^3 + 4x^2 + 3}$$

$$14.27 \text{ a) } \lim_{x \rightarrow \infty} \frac{x^4 - 3x^2 + 2}{5x^4 + 3x + 2}$$

$$14.28 \text{ a) } \lim_{x \rightarrow \infty} \frac{4x - 7x^2}{3x^2 - x + 2}$$

$$6) \lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{x + 3}$$

$$6) \lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - 3x - 4}$$

$$6) \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 6x + 4}$$

$$6) \lim_{x \rightarrow 1} \frac{2x^2 + 3x - 5}{3x - 3}$$

$$6) \lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{5x - x^2 - 6}$$

$$6) \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{-x^2 + 5x - 6}$$

$$6) \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^2 - 3x + 2}$$

$$6) \lim_{x \rightarrow 1} \frac{2x^2 + 3x - 5}{x^2 - 4x + 3}$$

$$6) \lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{7x - x^2 - 12}$$

$$6) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{7x - 3x^2 - 2}$$

$$6) \lim_{x \rightarrow -2} \frac{x^3 + 3x^2 + 2x}{x^2 - x - 6}$$

$$6) \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{3x^2 - x - 2}$$

$$6) \lim_{x \rightarrow 4} \frac{x^2 + 3x - 28}{x^2 - 4x}$$

$$6) \lim_{x \rightarrow 7} \frac{x^2 - 5x - 14}{2x^2 - 9x - 35}$$

$$6) \lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{3x^2 + 10x + 3}$$

$$6) \lim_{x \rightarrow 1} \frac{4x^2 + x - 5}{x^2 - 2x - 1}$$

$$6) \lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10}$$

$$6) \lim_{x \rightarrow 6} \frac{2x^2 - 9x - 18}{x^2 - 7x + 6}$$

$$B) \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+2x} - 1}$$

$$B) \lim_{x \rightarrow 2} \frac{\sqrt{2x} - x}{x - 2}$$

$$B) \lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{2x+3} - 3}$$

$$B) \lim_{x \rightarrow 0} \frac{2x^2}{\sqrt{x^2 + 2} - \sqrt{2}}$$

$$B) \lim_{x \rightarrow 0} \frac{\sqrt{5+x} - 3}{x^2 - 16}$$

$$B) \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - \sqrt{9-x}}{3x}$$

$$B) \lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 3} - 1}{x^2 - 4}$$

$$B) \lim_{x \rightarrow 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x}$$

$$B) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+8} - 3}$$

$$B) \lim_{x \rightarrow 0} \frac{\sqrt{9-x} - 3}{x^2 + x}$$

$$B) \lim_{x \rightarrow 0} \frac{2 - \sqrt{x^2 + 4}}{3x^2}$$

$$B) \lim_{x \rightarrow 9} \frac{\sqrt{2x+7} - 5}{3 - \sqrt{x}}$$

$$B) \lim_{x \rightarrow 0} \frac{2 - \sqrt{x}}{\sqrt{6x+1} - 5}$$

$$B) \lim_{x \rightarrow 0} \frac{3x}{\sqrt{10+x} - \sqrt{10-x}}$$

$$B) \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+2} - 2}$$

$$B) \lim_{x \rightarrow 2} \frac{\sqrt{4x+1} - 3}{x^3 - 8}$$

$$B) \lim_{x \rightarrow 0} \frac{\sqrt{4+3x} - \sqrt{4-3x}}{7x}$$

$$B) \lim_{x \rightarrow 3} \frac{\sqrt{4x-3} - 3}{x^2 - 9}$$

$$14.29 \text{ а) } \lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{6x^2 + 4x + 9}$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 + 6x - 16}$$

$$\text{в) } \lim_{x \rightarrow 2} \frac{1 - \sqrt{x^2 - 3}}{x^2 - 4}$$

$$14.30 \text{ а) } \lim_{x \rightarrow \infty} \frac{x^4 + 10x^2 - 3}{2x^5 - 5x^4 + 3x}$$

$$\text{б) } \lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{2x^2 + 5x - 7}$$

$$\text{в) } \lim_{x \rightarrow 0} \frac{3x^2}{\sqrt{1-x^2} - \sqrt{1+x^2}}$$

15. Найти область определения и точки разрыва функции $y = f(x)$, исследовать их характер.

$$15.1 \ y = \frac{x^3 + 4}{x^2}$$

$$15.2 \ y = \frac{x^2 - x + 1}{x - 1}$$

$$15.3 \ y = \frac{2}{x^2 + 2x}$$

$$15.4 \ y = \frac{4x^2}{x + 3}$$

$$15.5 \ y = \frac{12x}{x^2 - 9}$$

$$15.6 \ y = \frac{x^2 - 3x + 3}{x - 1}$$

$$15.7 \ y = \frac{4 - x^3}{x^2}$$

$$15.8 \ y = \frac{x^2 - 4x + 1}{x - 4}$$

$$15.9 \ y = \frac{x}{(1+x)^2}$$

$$15.10 \ y = \frac{x^2}{x^2 - 1}$$

$$15.11 \ y = \frac{4}{x^2 + 2x - 3}$$

$$15.12 \ y = \frac{\sqrt[3]{x^3 + 2}}{x}$$

$$15.13 \ y = \frac{e^{2-x}}{2-x}$$

$$15.14 \ y = \frac{x}{\sqrt[3]{1-x^2}}$$

$$15.15 \ y = \frac{x^2}{(x-1)^2}$$

$$15.16 \ y = \frac{1}{x} - \frac{1}{x+1}$$

$$15.17 \ y = \frac{1}{x-1} - \frac{1}{x}$$

$$15.18 \ y = \frac{4x^2 + 9}{4x + 8}$$

$$15.19 \ y = \left(\frac{x}{x+4} \right)^2$$

$$15.20 \ y = \frac{8x}{4-x^2}$$

$$15.21 \ y = \frac{e^{x+3}}{x+3}$$

$$15.22 \ y = \frac{16}{x^2(x-4)}$$

$$15.23 \ y = \frac{2}{x^2 + 2x}$$

$$15.24 \ y = \frac{x^2 - 6x + 3}{x - 3}$$

$$15.25 \ y = \frac{(x+1)^2}{x^2 + 2x}$$

$$15.26 \ y = \frac{x^3}{x^3 + 1}$$

$$15.27 \ y = \frac{x^3}{x^4 - 1}$$

$$15.28 \ y = \frac{x^3 - 3x}{x^2 - 1}$$

$$15.29 \ y = \frac{x^3 - 4}{2x}$$

$$15.30 \ y = \left(1 + \frac{1}{x} \right)^2$$

16. Найти производные функций, заданных явно.

$$16.1 \text{ а) } y = 3x^2 + \frac{2}{x^3} - \sqrt{2}$$

$$\text{б) } y = \frac{x^3}{4+x^2}$$

$$\text{в) } y = \sqrt[3]{x^2 + 4x + 5}$$

$$\text{г) } y = \frac{\arctg^3 4x}{\ln(6x-1)}$$

$$16.2 \text{ а) } y = 2x^3 + \sqrt[3]{x} - \ln 3$$

$$\text{б) } y = \frac{x^2 - x + 1}{x - 1}$$

$$\text{в) } y = (2x - 1) \cdot e^{-3x}$$

$$\text{г) } y = \frac{\sqrt{\arcsin 3x}}{\ln(5x-1)}$$

$$16.3 \text{ а) } y = 2x^4 + \sqrt[3]{x^2} - \sin 2$$

$$\text{б) } y = \frac{2}{x^2 + 2x}$$

$$\text{в) } y = \frac{2-x^2}{\sqrt{3+4x^2}}$$

$$\text{г) } y = \text{ctg} 3x \cdot \cos^4 7x$$

- 16.4 a) $y = x^2 - \frac{4}{\sqrt{x}} + \sqrt[3]{2}$ б) $y = \frac{4x^2}{3+x^2}$ в) $y = x^2 \cdot \sqrt{16-4x}$ г) $y = \frac{\cos^3 4x}{\sin 4x + 1}$
- 16.5 a) $y = \frac{4}{x} - x^3 - 2\cos 2$ б) $y = \frac{12x}{9+x^2}$ в) $y = \frac{2x+3}{\sqrt[4]{3x+5}}$ г) $y = (\cos 4x + 2)\sin 3x$
- 16.6 a) $y = \frac{4}{x^2} - 3x^2 - 6e^2$ б) $y = \frac{x^2 - 3x + 3}{x-1}$ в) $y = x^2 \cdot \ln(3x+1)$ г) $y = \frac{e^{-2x} + 3}{x^3 - 4x + 1}$
- 16.7 a) $y = 4x^4 + \sqrt[3]{x^2} - \ln 3$ б) $y = \frac{4-x^3}{x^2}$ в) $y = (2x+1)^3 \arctg 4x$ г) $y = \frac{\cos^3 4x}{\ln(5x+1)}$
- 16.8 a) $y = 3x^5 - \sqrt[3]{x^4} + \sin 4$ б) $y = \frac{x^2 - 4x + 1}{x-4}$ в) $y = (2x+3)\sqrt{4x-2}$ г) $y = \frac{e^{-3x^2}}{(3x-2)}$
- 16.9 a) $y = 2x^4 + \frac{4}{x^5} - \sqrt{3}$ б) $y = \frac{x^2}{2x^3+1}$ в) $y = e^{-4x} \cdot \ln(2x-4)$ г) $y = \frac{\sin^3 4x}{\sqrt[4]{5x-1}}$
- 16.10 a) $y = 3x^2 + 4\sqrt[3]{x^2} - \ln 9$ б) $y = \frac{x^2}{3x+4}$ в) $y = 3\sin^2 4x \cdot \tg 3x$ г) $y = \frac{\arcsin 4x}{\sqrt[3]{\arctg x}}$
- 16.11 a) $y = 2x^3 - \frac{5}{\sqrt{x^3}} + \sqrt[3]{6}$ б) $y = \frac{x^3}{x^2+6}$ в) $y = \sin 2x \cdot \sqrt{x^2+x}$ г) $y = \frac{\arctg^2 3x}{6x^3+1}$
- 16.12 a) $y = 2x^4 + \sqrt[5]{x} + \ln 4$ б) $y = \frac{4x^3-3x}{4x^2-1}$ в) $y = (x^2+1) \cdot e^{3x}$ г) $y = \frac{\sqrt{\arcsin 2x}}{x^4}$
- 16.13 a) $y = 2x^5 + \frac{3}{x^4} - \sqrt[3]{2}$ б) $y = \frac{3x^2-7}{2x+1}$ в) $y = \frac{\sin 2x}{\sqrt[3]{x^2-1}}$ г) $y = \tg 3x \cdot \ln^2 4x$
- 16.14 a) $y = 6x^3 - \frac{3}{\sqrt{x^5}} + \sqrt[3]{4}$ б) $y = \frac{17-x^2}{4x-5}$ в) $y = \cos \sqrt{1-\ln^2 x}$ г) $y = \frac{x^3}{\sqrt{x^4+1}}$
- 16.15 a) $y = \frac{4}{x^2} - 2x^4 + \cos 3$ б) $y = \frac{x^2+1}{4x^2-3}$ в) $y = \frac{\sqrt{2x-1}}{(x+1)^2}$ г) $y = \sqrt{x^2+1} \cdot \cos 6x$
- 16.16 a) $y = \frac{3}{x^4} + 3\sqrt{x} - 3e^4$ б) $y = \frac{x^3-4x}{3x^2-4}$ в) $y = x^3 \cdot \ln \sqrt{2x+x^2}$ г) $y = \frac{e^{\sqrt{x}}}{x^4+1}$
- 16.17 a) $y = 6x^3 + \sqrt[3]{x^4} - \ln 5$ б) $y = \frac{4x^2+9}{4x+8}$ в) $y = (x+1)^2 \cdot \arctg 2x$ г) $y = \frac{\cos^2 3x}{\ln \sqrt{x}}$
- 16.18 a) $y = 3x^4 - \sqrt[3]{x^5} + \sin 6$ б) $y = \frac{x^2-3}{3x^2-2}$ в) $y = \sqrt{x^3} \cdot \sin \sqrt{x+2}$ г) $y = \frac{\sin^2 x}{1+\tg x}$
- 16.19 a) $y = 2x^6 + \frac{3}{x^4} - \sqrt{5}$ б) $y = \frac{2x^2-6}{x-2}$ в) $y = e^{x^3} \cdot \ln(x^2+4)$ г) $y = \arctg \left(\frac{1}{x^2+1} \right)$
- 16.20 a) $y = 4x^5 + 2\sqrt[4]{x^3} - \ln 3$ б) $y = \frac{x^3-5x}{5-3x^2}$ в) $y = \cos^2 3x \cdot \ln 3x$ г) $y = \frac{\sin 4x}{\sqrt[3]{x^2+1}}$

16.21 а) $y = 2x^6 - \frac{4}{\sqrt{x^5}} + \sqrt[3]{3}$ б) $y = \frac{2-x^2}{9x^2-4}$ в) $y = \operatorname{tg} 2x \cdot \sqrt{3x+4}$ г) $y = \frac{\ln^2 3x}{x + \cos 2x}$

16.22 а) $y = 2x^6 + \sqrt[5]{x^2} + \ln 2$ б) $y = \frac{x^2+16}{9x^2-8}$ в) $y = (x^2+2x) \cdot e^{-3x^2}$ г) $y = \frac{\sqrt{\sin 4x}}{(1+x^2)^3}$

16.23 а) $y = 4x^5 + \frac{3}{x^6} - \sqrt[5]{2}$ б) $y = \frac{21-x^2}{7x+9}$ в) $y = \frac{\ln \sin x}{\sqrt[3]{x^3+1}}$ г) $y = \operatorname{tg}(e^x) \cdot (4x+3)^2$

16.24 а) $y = 3x^6 - \frac{3}{\sqrt[3]{x^5}} + \sqrt[3]{2}$ б) $y = \frac{2x^2-1}{x^2-2}$ в) $y = \sin(3x+2) \cdot e^{\cos x}$ г) $y = \frac{\arcsin(x^3)}{\sqrt{x^2+1}}$

16.25 а) $y = \frac{4}{x^3} - 3x^8 + \cos 2$ б) $y = \frac{x^2-11}{4x-3}$ в) $y = \frac{\sqrt{\operatorname{tg} x}}{\sin(e^{2x})}$ г) $y = (x^2+2\sqrt{x}) \cdot \operatorname{arctg} 3x$

16.26 а) $y = \frac{4}{x^5} + 5\sqrt{x^3} - 4e^3$ б) $y = \frac{x^2}{1-2x^3}$ в) $y = \sqrt{1+\cos^3 2x}$ г) $y = \operatorname{arctg} \frac{x}{\sqrt{x^2+1}}$

16.27 а) $y = 6x^5 + \sqrt[4]{x^7} - \ln 3$ б) $y = \frac{2x^2-9}{x^2-1}$ в) $y = \ln \frac{\sqrt{x^2+x+1}}{x}$ г) $y = e^{\sin 2x} \cdot \operatorname{arctg} \sqrt{x}$

16.28 а) $y = 5x^6 - \sqrt[4]{x^5} + \sin 3$ б) $y = \frac{x^2+2x}{2x+1}$ в) $y = \arcsin \frac{x}{x+1}$ г) $y = \frac{\operatorname{tg}(\ln x)}{\sqrt{2x+3}}$

16.29 а) $y = 6x^5 + \frac{4}{x^8} - \sqrt{2}$ б) $y = \frac{3x^2-10}{4x^2-1}$ в) $y = \operatorname{arctg} \sqrt{\frac{1-x}{1+x}}$ г) $y = e^{\cos x} \cdot \operatorname{arctg} \left(\frac{1}{\sqrt{x}} \right)$

16.30 а) $y = 2x^8 + 2\sqrt[4]{x^5} - \ln 6$ б) $y = \frac{x^2}{x^3-2}$ в) $y = \ln \sqrt{\frac{e^x}{1+e^x}}$ г) $y = \sin(\ln x) \cdot (x^2+2x)^3$

17. Найти производную функции, заданной параметрически.

17.1 $\begin{cases} x = \sin 2t \\ y = \cos 2t \end{cases}$ 17.2 $\begin{cases} x = \operatorname{tg} 3t \\ y = \operatorname{ctg} 3t \end{cases}$ 17.3 $\begin{cases} x = (2t-1)/t \\ y = \sqrt{t^3} \end{cases}$

17.4 $\begin{cases} x = \ln^2 4t \\ y = \sqrt{\ln t} \end{cases}$ 17.5 $\begin{cases} x = \sin 3t \\ y = \operatorname{ctg} 3t \end{cases}$ 17.6 $\begin{cases} x = \operatorname{tg}^2 t \\ y = \sin 2t \end{cases}$

17.7 $\begin{cases} x = \operatorname{tg} 4t \\ y = \operatorname{ctg} 2t \end{cases}$ 17.8 $\begin{cases} x = 3\sin 2t \\ y = \cos 4t \end{cases}$ 17.9 $\begin{cases} x = \arcsin 2t \\ y = \operatorname{arctg} 2t \end{cases}$

17.10 $\begin{cases} x = \ln 5t \\ y = \lg 3t \end{cases}$ 17.11 $\begin{cases} x = e^{\sin t} \\ y = e^{\cos t} \end{cases}$ 17.12 $\begin{cases} x = \sqrt{\sin t} \\ y = \cos \sqrt{t} \end{cases}$

17.13 $\begin{cases} x = \ln(t^2+1) \\ y = 1 + \ln^2 t \end{cases}$ 17.14 $\begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}$ 17.15 $\begin{cases} x = \frac{t+1}{t} \\ y = \frac{t-1}{t} \end{cases}$

$$17.16 \begin{cases} x = \frac{1}{1+4t^2} \\ y = \arctg 2t \end{cases}$$

$$17.17 \begin{cases} x = \cos^2 t \\ y = \ln \sin t \end{cases}$$

$$17.18 \begin{cases} x = 2t^3 + t \\ y = \ln t \end{cases}$$

$$17.19 \begin{cases} x = \frac{1}{2+t} \\ y = \ln(2t+t^2) \end{cases}$$

$$17.20 \begin{cases} x = \ln \cos t \\ y = e^{\sin t} \end{cases}$$

$$17.21 \begin{cases} x = \frac{\cos t}{1 + \cos t} \\ y = \frac{\sin t}{1 + \cos t} \end{cases}$$

$$17.22 \begin{cases} x = t\sqrt{1-t^2} \\ y = \arcsin t \end{cases}$$

$$17.23 \begin{cases} x = \frac{t^2+1}{t^3} \\ y = \sin(t^2+t) \end{cases}$$

$$17.24 \begin{cases} x = \ln(\operatorname{tg} t) \\ y = \frac{1}{\sin^2 t} \end{cases}$$

$$17.25 \begin{cases} x = \sqrt{1-t^2} \\ y = \frac{t}{\sqrt{1-t^2}} \end{cases}$$

$$17.26 \begin{cases} x = \arcsin \sqrt{t} \\ y = \sqrt{1+\sqrt{t}} \end{cases}$$

$$17.27 \begin{cases} x = \arctg t \\ y = \ln \left(\frac{\sqrt{1+t^2}}{t+1} \right) \end{cases}$$

$$17.28 \begin{cases} x = \arctg(e^t) \\ y = \sqrt{e^{2t}+1} \end{cases}$$

$$17.29 \begin{cases} x = \ln(1-t^2) \\ y = \arcsin \sqrt{1-t^2} \end{cases}$$

$$17.30 \begin{cases} x = \ln \left(\frac{1-t}{1+t} \right) \\ y = \sqrt{1-t^2} \end{cases}$$

18. Вычислить пределы, используя правило Лопиталя.

$$18.1 \lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{2x^2 + 5x - 7}$$

$$18.2 \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x^2 + 6x - 16}$$

$$18.3 \lim_{x \rightarrow 6} \frac{2x^2 - 9x - 18}{x^2 - 7x + 6}$$

$$18.4 \lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10}$$

$$18.5 \lim_{x \rightarrow 1} \frac{4x^2 + x - 5}{x^2 - 2x - 1}$$

$$18.6 \lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{3x^2 + 10x + 3}$$

$$18.7 \lim_{x \rightarrow 7} \frac{x^2 - 5x - 14}{2x^2 - 9x - 35}$$

$$18.8 \lim_{x \rightarrow 4} \frac{x^2 + 3x - 28}{x^2 - 4x}$$

$$18.9 \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{3x^2 - x - 2}$$

$$18.10 \lim_{x \rightarrow -2} \frac{x^3 + 3x^2 + 2x}{x^2 - x - 6}$$

$$18.11 \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{7x - 3x^2 - 2}$$

$$18.12 \lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{7x - x^2 - 12}$$

$$18.13 \lim_{x \rightarrow 1} \frac{2x^2 + 3x - 5}{x^2 - 4x + 3}$$

$$18.14 \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^2 - 3x + 2}$$

$$18.15 \lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{-x^2 + 5x - 6}$$

$$18.16 \lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{5x - x^2 - 6}$$

$$18.17 \lim_{x \rightarrow 1} \frac{2x^2 + 3x - 5}{3x - 3}$$

$$18.18 \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2x^2 - 6x + 4}$$

$$18.19 \lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - 3x - 4}$$

$$18.20 \lim_{x \rightarrow -3} \frac{2x^2 + 5x - 3}{x + 3}$$

$$18.21 \lim_{x \rightarrow 6} \frac{x^2 - 9x + 18}{3x^2 - 17x - 6}$$

$$18.22 \lim_{x \rightarrow 6} \frac{2x^2 - 9x - 18}{x^2 - 7x + 6}$$

$$18.23 \lim_{x \rightarrow 5} \frac{2x^2 - 11x + 5}{x^2 - 7x + 10}$$

$$18.24 \lim_{x \rightarrow 7} \frac{2x^2 - 13x - 7}{x^2 - 9x + 14}$$

$$18.25 \lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1}{2x^2 + 5x - 7}$$

$$18.26 \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 5x + 6}$$

$$18.27 \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{3x^2 + 4x - 7}$$

$$18.28 \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{3x^2 - 5x - 2}$$

$$18.29 \lim_{x \rightarrow 3} \frac{3x^2 - 10x + 3}{x^2 - 2x - 3}$$

$$18.30 \lim_{x \rightarrow -1} \frac{2x^2 + 5x + 3}{x^2 - 4x - 5}$$

19. Составить уравнения касательной к графику функции $y = f(x)$ в точке с абсциссой x_0 .

$$19.1 y = \frac{1}{x^2 + 1}, \quad x_0 = 1$$

$$19.2 y = \frac{1}{x^3 + 1}, \quad x_0 = 0$$

$$19.3 y = x + \frac{1}{x}, \quad x_0 = 1$$

$$19.4 y = \frac{(4x - x^2)}{4}, \quad x_0 = 2$$

$$19.5 y = 2x^2 + 3x - 1, \quad x_0 = -2$$

$$19.6 y = x - x^3, \quad x_0 = -1$$

$$19.7 y = x^2 + 8\sqrt{x} - 32, \quad x_0 = 4$$

$$19.8 y = x + x\sqrt{x}, \quad x_0 = 1$$

$$19.9 y = 4x - x^2, \quad x_0 = 3$$

$$19.10 y = \sqrt{4 - 2x^2}, \quad x_0 = 1$$

$$19.11 y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}, \quad x_0 = 4$$

$$19.12 y = \frac{x^2 - 3x + 6}{x^2}, \quad x_0 = 3$$

$$19.13 y = 2x^2 - 3x + 1, \quad x_0 = 1$$

$$19.14 y = \sqrt{x} - 3 \cdot \sqrt[3]{x}, \quad x_0 = 64$$

$$19.15 y = x^2 + \sqrt{x^3}, \quad x_0 = 1$$

$$19.16 y = x^2 + x + 1, \quad x_0 = -1$$

$$19.17 y = \frac{2 + \sqrt{x}}{2 - \sqrt{x}}, \quad x_0 = 9$$

$$19.18 y = \frac{x^2 - 2x + 2}{x^2}, \quad x_0 = 2$$

$$19.19 y = 2x^2 + 3, \quad x_0 = -1$$

$$19.20 y = 3 \cdot \sqrt[4]{x} - \sqrt{x}, \quad x_0 = 1$$

$$19.21 y = 2x + \frac{1}{x}, \quad x_0 = 1$$

$$19.22 y = \frac{x^3 + 2}{x^3 - 2}, \quad x_0 = 2$$

$$19.23 y = x^2 - x - 1, \quad x_0 = 1$$

$$19.24 y = 3 \cdot \sqrt[3]{x^2} - 2x + 2, \quad x_0 = 1$$

$$19.25 y = \frac{1}{3x + 2}, \quad x_0 = 2$$

$$19.26 y = \frac{2x}{x^2 + 1}, \quad x_0 = 1$$

$$19.27 y = \frac{x^2 - 3x + 3}{3}, \quad x_0 = 3$$

$$19.28 y = \frac{1 + 3x^2}{3 + x^2}, \quad x_0 = 1$$

$$19.29 y = \frac{3x - 2x^3}{3}, \quad x_0 = 1$$

$$19.30 y = \frac{x^2 - 2x - 3}{4}, \quad x_0 = 4$$

20. Найти наибольшее и наименьшее значения функции на отрезке $[a, b]$.

$$20.1 y = x^3 - 3x^2 + 4 \quad a = 0, \quad b = 4$$

$$20.2 y = x^2 + \frac{16}{x} - 16 \quad a = 1, \quad b = 4$$

$$20.3 y = x^3 - 2x^2 + 6 \quad a = 0, \quad b = 3$$

$$20.4 y = 4 - x - \frac{4}{x^2} \quad a = 1, \quad b = 4$$

$$20.5 y = x^3 - 5x^2 + 4 \quad a = 1, \quad b = 4$$

$$20.6 y = 2\sqrt{x} - x \quad a = 0, \quad b = 4$$

$$20.7 \ y = \frac{10x+10}{x^2+2x+2} \quad a=-1, \ b=2$$

$$20.8 \ y = 1 - \sqrt[3]{x^2-2x} \quad a=0, \ b=2$$

$$20.9 \ y = x^3 - 3x^2 + 6 \quad a=1, \ b=4$$

$$20.10 \ y = 2x - 3 \cdot \sqrt[3]{x^2} \quad a=-1, \ b=1$$

$$20.11 \ y = x^3 - 7x^2 + 2 \quad a=2, \ b=5$$

$$20.12 \ y = x^2(x-2)^2 \quad a=0, \ b=2$$

$$20.13 \ y = 1 + \sqrt[3]{x^2+2x} \quad a=-2, \ b=0$$

$$20.14 \ y = x - 4\sqrt{x} + 5 \quad a=1, \ b=9$$

$$20.15 \ y = \frac{10x}{1+x^2} \quad a=0, \ b=3$$

$$20.16 \ y = 3 - x - \frac{4}{(x+2)^2} \quad a=-1, \ b=2$$

$$20.17 \ y = x^3 - 2x^2 + 4 \quad a=0, \ b=3$$

$$20.18 \ y = x - 4\sqrt{x+2} + 8 \quad a=-1, \ b=7$$

$$20.19 \ y = x^3 - 4x^2 + 6 \quad a=1, \ b=4$$

$$20.20 \ y = \frac{x^3-9x^2}{4} + 6x - 9 \quad a=0, \ b=4$$

$$20.21 \ y = x^3 - 4x^2 + 5 \quad a=1, \ b=3$$

$$20.22 \ y = \frac{4x}{4+x^2} \quad a=-4, \ b=2$$

$$20.23 \ y = x^3 - 6x^2 + 6 \quad a=2, \ b=4$$

$$20.24 \ y = \frac{4x^2}{3+x^2} \quad a=-1, \ b=1$$

$$20.25 \ y = x^3 - 2x^2 + 2 \quad a=0, \ b=3$$

$$20.26 \ y = -\frac{x^2}{2} + \frac{8}{x} + 8 \quad a=-4, \ b=-1$$

$$20.27 \ y = 2x^3 - 3x^2 - 4 \quad a=0, \ b=2$$

$$20.28 \ y = 2\sqrt{x-1} - x + 2 \quad a=1, \ b=5$$

$$20.29 \ y = 2 - 12x^2 - 8x^3 \quad a=-2, \ b=0$$

$$20.30 \ y = 3\sqrt[3]{(x-3)^2} - 2x + 6 \quad a=2, \ b=4$$

21. Найти локальные экстремумы, интервалы возрастания и убывания функции $y = f(x)$.

$$21.1 \ y = \ln(4-x^2)$$

$$21.2 \ y = xe^{-x}$$

$$21.3 \ y = \sqrt{x} + \sqrt{4-x}$$

$$21.4 \ y = (2+x^2) \cdot e^{-x^2}$$

$$21.5 \ y = \frac{x^3}{\sqrt{x^4+1}}$$

$$21.6 \ y = \frac{x}{\sqrt[3]{(x-2)^2}}$$

$$21.7 \ y = \arctg(x^2)$$

$$21.8 \ y = \sqrt[3]{x^2-2x}$$

$$21.9 \ y = x\sqrt{x+3}$$

$$21.10 \ y = x + 2\arctg x$$

$$21.11 \ y = \sqrt[3]{1-x^2}$$

$$21.12 \ y = \sqrt[3]{1-x^3}$$

$$21.13 \ y = \frac{e^x}{1+x}$$

$$21.14 \ y = \frac{x}{\sqrt{x^2+1}}$$

$$21.15 \ y = \frac{x^2}{\sqrt{x^2+1}}$$

$$21.16 \ y = \ln \left(\frac{\sqrt{x^2+1}-1}{x} \right)$$

$$21.17 \ y = \frac{x}{\ln x}$$

$$21.18 \ y = \frac{4}{\sqrt{4-x^2}}$$

$$21.19 \ y = \frac{8}{x\sqrt{x^2-4}}$$

$$21.20 \ y = \ln(x^2+2x+2)$$

$$21.21 \ y = \frac{\ln x}{x}$$

$$21.22 \ y = \frac{1}{x \ln x}$$

$$21.23 \ y = x^2 e^{2/x}$$

$$21.24 \ y = \frac{1}{e^x - 1}$$

$$21.25 \ y = x - 2 \ln x$$

$$21.26 \ y = x e^{-x^2/2}$$

$$21.27 \ y = \ln \left(\frac{x}{x-1} \right)$$

$$21.28 \ y = \frac{1 + \sqrt{x}}{3 + x}$$

$$21.29 \ y = (x-1)e^x$$

$$21.30 \ y = \frac{1}{x} \cdot e^{1/x}$$

22. Провести полное исследование и построить график неперiodической функции

$y = f(x)$:

- 1) найти область определения функции;
- 2) найти точки разрыва;
- 3) исследовать функцию на чётность, нечётность;
- 4) найти точки пересечения графика с осями координат;
- 5) найти асимптоты графика функции;
- 6) найти интервалы возрастания и убывания, локальные экстремумы функции;
- 7) найти интервалы выпуклости, вогнутости и точки перегиба;
- 8) построить график.

$$22.1 \text{ а) } y = x^3 - 3x^2; \text{ б) } y = \frac{x^2 - x - 6}{x - 2}.$$

$$22.2 \text{ а) } y = \frac{6x^2 - x^4}{9}; \text{ б) } y = \frac{x^2}{2} + \frac{1}{x}.$$

$$22.3 \text{ а) } y = (x-1)^2 \cdot (x+2); \text{ б) } y = x^2 + \frac{2}{x}.$$

$$22.4 \text{ а) } y = \frac{(x-2)^2 \cdot (x+4)}{4}; \text{ б) } y = \frac{8}{x^2 - 4}.$$

$$22.5 \text{ а) } y = x^4 - 2x^2 + 3; \text{ б) } y = \frac{16}{x^2(x-4)}.$$

$$22.6 \text{ а) } y = 2x^3 - 9x^2 + 12x - 9; \text{ б) } y = \frac{x^3}{x^4 - 1}.$$

$$22.7 \text{ а) } y = \frac{1}{3} \cdot x^3 - x^2 + 4; \text{ б) } y = \frac{4x - 12}{(x-2)^2}.$$

$$22.8 \text{ а) } y = 2x^3 - 3x^2 - 4; \text{ б) } y = \frac{x^3}{x^3 + 1}.$$

$$22.9 \text{ а) } y = 2x^3 + 3x^2 - 5; \text{ б) } y = \frac{1 - x^3}{x^2}.$$

$$22.10 \text{ а) } y = 12x^2 - 8x^3 - 2; \text{ б) } y = \frac{x^2 - 1}{x^2 + 1}.$$

$$22.11 \text{ а) } y = \frac{4x^3 - x^4}{5}; \text{ б) } y = \frac{x^3 - 3x}{x^2 - 1}.$$

$$22.12 \text{ а) } y = 3x^2 - 2 - x^3; \text{ б) } y = \frac{x}{2 - x^3}.$$

$$22.13 \text{ а) } y = (x-1)^2(x-3)^2; \text{ б) } y = \frac{x^2}{x^3 - 1}.$$

$$22.14 \text{ а) } y = x^3 - 6x^2 + 9x - 3; \text{ б) } y = \frac{x}{x^2 - 1}.$$

$$22.15 \text{ а) } y = x^4 - 5x^2 + 4; \text{ б) } y = \frac{x}{x^3 + 2}.$$

$$22.16 \text{ а) } y = x^3 + 6x^2 + 9x + 8; \text{ б) } y = \frac{x^2 - 6x + 3}{x - 3}.$$

$$22.17 \text{ а) } y = -x^4 + 8x^2 + 9; \text{ б) } y = \frac{x^3}{x^2 - 3}.$$

$$22.18 \text{ а) } y = 16x^3 + 12x^2 - 5; \text{ б) } y = \frac{4}{x^2 + 2x - 3}.$$

$$22.19 \text{ а) } y = x^3 - 6x^2 + 9x - 4; \text{ б) } y = \frac{x^3}{2(x-1)^2}.$$

$$22.20 \text{ а) } y = x^2(x-2)^2; \text{ б) } y = \frac{x}{(1-x^2)^2}.$$

$$22.21 \text{ а) } y = 2x^3 - 3x^2 - 12x - 1; \text{ б) } y = \frac{x^4}{x^3 - 1}.$$

$$22.22 \text{ а) } y = (1-x^2)^3; \text{ б) } y = \frac{4x}{4+x^2}.$$

$$22.23 \text{ а) } y = \frac{(x^2 - 4)^2}{16}; \text{ б) } y = \frac{4x^2}{3 + x^2}.$$

$$22.24 \text{ а) } y = x^3 - 6x^2 + 16; \text{ б) } y = \frac{x^2 - 4x + 1}{x - 4}.$$

$$22.25 \text{ а) } y = x^3 - 3x^2 - 9x + 10; \text{ б) } y = \frac{x^4 - 3}{x}.$$

$$22.26 \text{ а) } y = x^3 - 12x; \text{ б) } y = \frac{x^2 - 6x + 9}{(x-1)^2}.$$

$$22.27 \text{ а) } y = \frac{(x^2 - 5)^3}{125}; \text{ б) } y = \frac{x^4}{x^3 + 1}.$$

$$22.28 \text{ а) } y = \frac{x^3}{3} - x^2 + 4; \text{ б) } y = \frac{(x-1)^2}{x^2 + 1}.$$

$$22.29 \text{ а) } y = 3x^5 - 5x^4 + 4; \text{ б) } y = \frac{3x-2}{x^3}.$$

$$22.30 \text{ а) } y = 2x + \frac{x^2}{2} - \frac{x^3}{3}; \text{ б) } y = \frac{x^2 - 2x + 2}{x-1}.$$

23. Найти частные производные: $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ и первый дифференциал dz функции

$$z = f(x, y).$$

$$23.1 \quad z = \sqrt{2xy + y^2}$$

$$23.2 \quad z = \ln(x^2 + 2y)$$

$$23.3 \quad z = \frac{y}{x} - \frac{x}{y}$$

$$23.4 \quad z = y \sin(x^2)$$

$$23.5 \quad z = x \cos 2y$$

$$23.6 \quad z = \frac{\cos(xy)}{y}$$

$$23.7 \quad z = \sin(3x + 2y)$$

$$23.8 \quad z = \cos(2x + 3y)$$

$$23.9 \quad z = \frac{\sin(xy)}{x}$$

$$23.10 \quad z = \ln(5x + 4y)$$

$$23.11 \quad z = x \sin^2 y$$

$$23.12 \quad z = \ln(x + y^2)$$

$$23.13 \quad z = e^{3x-2y}$$

$$23.14 \quad z = e^{x^3 y^2}$$

$$23.15 \quad z = \sin(x^2 + y^3)$$

$$23.16 \quad z = \frac{x}{x+y}$$

$$23.17 \quad z = \ln(3x + y^2)$$

$$23.18 \quad z = \sin 2x \cdot \cos 3y$$

$$23.19 \quad z = \sqrt{x^3 + y^2}$$

$$23.20 \quad z = \arctg(x^2 \cdot y)$$

$$23.21 \quad y = x \ln \left(\frac{x}{y} \right)$$

$$23.22 \quad z = e^{2x} \cos 3y$$

$$23.23 \quad z = \arctg \left(\frac{x}{y} \right)$$

$$23.24 \quad z = \arcsin \left(\frac{y}{x} \right)$$

$$23.25 \quad z = \arcsin(x \cdot y^2)$$

$$23.26 \quad z = \ln(x + \ln y)$$

$$23.27 \quad z = \sin \left(\frac{y}{x^2} \right)$$

$$23.28 \quad z = \arctg(x^2 + y)$$

$$23.29 \quad z = \cos \left(\frac{y}{\sqrt{x}} \right)$$

$$23.30 \quad z = \ln \left(\frac{x+y}{y} \right)$$

24. Найти градиент функции $\text{grad} u$ и его модуль $|\text{grad} u|$ в точке $M(x_0, y_0, z_0)$.

$$24.1 \quad u = x + \ln(z^2 + y^2), \quad M(2, 1, 1).$$

$$24.2 \quad u = y \ln(1 + x^2) - \arctg z, \quad M(0, 1, 1).$$

$$24.3 \quad u = \ln(3 - x^2) + xy^2z, \quad M(1, 3, 2).$$

$$24.4 \quad u = xy - \frac{x}{z}, \quad M(-4, 3, -1).$$

$$24.5 \quad u = z^2 + 2\arctg(x - y), \quad M(1, 2, -1).$$

$$24.6 \quad u = \sqrt{xy} + \sqrt{9 - z^2}, \quad M(1, 1, 0).$$

$$24.7 \quad u = \ln(x + \sqrt{y^2 + z^2}), \quad M(1, -3, 4).$$

$$24.8 \quad u = x^2 y^2 z - \ln(z - 1), \quad M(1, 1, 2).$$

$$24.9 \quad u = \ln(x^2 + y^2) + xyz, \quad M(1, -1, 2).$$

$$24.10 \quad u = (x^2 + y^2 + z^2)^{3/2}, \quad M(1, 1, 1).$$

$$24.11 \quad u = 4\ln(3 + x^2) - 8xyz, \quad M(1, 1, 1).$$

$$24.12 \quad u = x\sqrt{y} + y\sqrt{z}, \quad M(2, 4, 4).$$

$$24.13 \quad u = -2\ln(x^2 - 5) - 4xyz, \quad M(1, 1, 1).$$

$$24.14 \quad u = \frac{x^2 y}{4} - \sqrt{x^2 + 5z^2}, \quad M(-2, \frac{1}{2}, 1).$$

$$24.15. u = x^2 y - \sqrt{xy + z^2}, M(1,5,-2).$$

$$24.17. u = xz^2 - \sqrt{x^3 y}, M(2,2,4).$$

$$24.19. u = x^2 - \arctg(y+z), M(2,1,1).$$

$$24.21. u = \arctg(y/x) + xz, M(2,2,-1).$$

$$24.23. u = \sqrt{x^2 + y^2} - z, M(3,4,1).$$

$$24.25. u = 2\sqrt{x+y} + y \arctg z, M(3,-2,1).$$

$$24.27. u = x\sqrt{y} - (z+y)\sqrt{x}, M(1,1,-2).$$

$$24.29. u = \sqrt{x^2 + 2y^2 + 3z^2}, M(-1,2,0).$$

$$24.16. u = x(\ln y - \arctg z), M(-2,1,-1).$$

$$24.18. u = x^3 + \sqrt{y^2 + z^2}, M(1,-3,4).$$

$$24.20. u = x\sqrt{y} - yz^2, M(2,1,-1).$$

$$24.22. u = \ln(1+x^2) - xy\sqrt{z}, M(1,-2,4).$$

$$24.24. u = \frac{\sqrt{x}}{y} - \frac{y \cdot z}{x + \sqrt{y}}, M(4,1,-2).$$

$$24.26. u = \sqrt{xy} + \sqrt{4-z^2}, M(1,1,0).$$

$$24.28. u = \arctg \left(\frac{z}{\sqrt{x^2 + y^2}} \right), M(1,1,1).$$

$$24.30. u = \ln(1+x^2+y^2) - \sqrt{x^2+z^2}, M(3,0,-4).$$

25. Найти локальные экстремумы функции $z = f(x, y)$:

$$25.1. z = x^2 + xy + y^2 - 12x - 3y$$

$$25.3. z = x^2 - xy + y^2 + 9x - 6y + 20$$

$$25.5. z = x^2 + y^2 - xy + 2x - y$$

$$25.7. z = 2x^2 + y^2 - x + 2y + 1$$

$$25.9. z = x^2 + y^2 + xy - 2x - 2y + 2$$

$$25.11. z = x^2 - xy + y^2.$$

$$25.13. z = x^2 - xy - y^2$$

$$25.15. z = x^2 + y^2 + xy - 3x - 6y$$

$$25.17. z = x^2 - 2xy + 4y^2 + 4$$

$$25.19. z = 3x^2 + y^2 - x + y$$

$$25.21. z = x^2 + y^2 + 2x + 4y - 6$$

$$25.22. z = 2xy - 2x - 4y$$

$$25.25. z = x^2 + y^2 + xy - 6x - 9y$$

$$25.27. z = 7x^2 + 3y^2 - 6xy - 4x + 7y$$

$$25.29. z = x^2 + 3y^2 - 6xy - 4x + 7y - 12$$

$$25.2. z = x^2 + 2y^2 - xy$$

$$25.4. z = x^2 + y^2 - xy + 2x - 2y + 1$$

$$25.6. z = 8x^2 + 3y^2 - 6xy$$

$$25.8. z = x^2 - 2xy + 2y^2 + 2x$$

$$25.10. z = x^2 + (y-1)^2$$

$$25.12. z = 4x + 2y - x^2 - y^2.$$

$$25.14. z = x^2 - x - y^2 + y - 18$$

$$25.16. z = 3 + 2x - y - x^2 + xy - y^2$$

$$25.18. z = 3xy + 2x - y^2 + 8$$

$$25.20. z = x^2 - xy + y^2 - 2x + y - 3$$

$$25.22. z = 2x^2 - x + (y+1)^2$$

$$25.24. z = x^2 + y^2 - xy + 3x - 2y$$

$$25.26. z = -x^2 + y^2 - xy + 3x + 6y$$

$$25.28. z = 3x^2 + 18xy - 8x + 18y + 8$$

$$25.30. z = 3x^2 - y^2 + 5xy - 6$$