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On Absolute Convergence of Multiple Fourier Series

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Abstract—We consider absolute convergence of multiple series of Fourier–Haar coefficients for functions of many variables with partial derivatives of higher order. We show that the obtained results are best possible for general orthonormal systems.

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1. AUXILIARY NOTATIONS AND RESULTS

Denote an element of an s-dimensional space R^s by $X = (x_1, \dots, x_s)$, $H = (h_1, \dots, h_s), \dots$ By $D_s = [0,1]^s$, denote a unit cube from R^s. If $X = (x_1, \dots, x_s)$ and $H = (h_1, \dots, h_s)$, then $X + H = (x_1 + h_1, \dots, x_s + h_s)$. Further, $\|H\| = \Big(\sum\limits_{k=1}^{s} h_k^2\Big)^{\frac{1}{2}}$.

Let $B \subset \{1, \dots, s\} = \overline{S}$, by X_B we denote a vector $(x_1', \dots, x_s')$, where $x_i' = x_i$, if $i \in B$, and $x_i' = 0$, if $i \notin B$. In addition, $B' = \overline{S} \setminus B$, Π_s is the set of all nonempty subsets of $\overline{S}$; $|B|$ is the number of elements of the set B, and $E = (1, \dots, 1)$ is the s-dimensional vector with all the coordinates equal one.

By $M = (m_1, \dots, m_s)$ and $N = (n_1, \dots, n_s)$, we denote the points from R^s with integer coordinates, $M \geq N$ means $m_i \geq n_i$, $i = 1, \dots, s$.

If $B = \{i_1, \dots, i_B\}$, then

$$\sum_{\nu_B = M_B}^{N_B} C_{\nu_B} = \sum_{\nu_{i_1} = m_{i_1}}^{n_{i_1}} \cdots \sum_{\nu_{i_{|B|}} = m_{i_{|B|}}}^{n_{i_{|B|}}} c_{\nu_1 \cdots \nu_{|B|}}.$$

It is said that $f(X) \in \operatorname{Lip}\alpha$ ($\alpha \in (0,1]$), if for $i = 1, \dots, s$

$$\sup_{|t_i| \leq h_i} \|f(X+T) - f(X)\|_{C(D_s)} = O(\|H\|^\alpha),$$

where $X \in D_s, T \in D_s, X + T \in D_s, H \in D_s$, and $C(D_s)$ is the space of all continuous on D_s functions with the norm $\|f\|_{C(D_s)} = \max\limits_{X \in D_s} |f(X)|$.

Suppose $H_{\{j\}} = (0, \dots, 0, h_j, 0, \dots, 0)$,

$$\Delta_{h_j}(f) = f(X + H_{\{j\}}) - f(X), \quad X \in D_s, \quad X + H_{\{j\}} \in D_s,$$

and $\Delta_{h_1 \cdots h_s}$ stands for iterated application of the operations Δ_{h_j}, as j ranges through the set $\overline{S}$.

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