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Method of Decreasing the Order of a Partial Differential Equation by Reducing to Two Ordinary Differential Equations

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Abstract—Using additional unknown functions and additional boundary conditions in the integral method of heat balance, we obtain approximate analytic solutions to the non-stationary thermal conductivity problem for an infinite solid cylinder that allow to estimate the temperature state practically in the whole time range of the non-stationary process. The thermal conducting process is divided into two stages with respect to time. The initial problem for the partial differential equation is represented in the form of two problems, in which the integration is performed over ordinary differential equations with respect to corresponding additional unknown functions. This method allows to simplify substantially the solving process of the initial problem by reducing it to the sequential solution of two problems, in each of them additional boundary conditions are used.

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The complexity of obtaining exact analytic solutions to boundary-value problems of non-stationary thermal conductivity for solids with central symmetry (a cylinder, a ball) is that the volume, into which the heat diffuses, does not remain the same for equal values of the radius increment, as is the case for plane problems. The applying of classical analytical methods and methods of integral transformations yield solutions in the form of infinite series containing the first-type Bessel functions of zero and first orders. The disadvantage of using Bessel functions in analytic solutions is that they have the form of infinite power series bounded by the number of terms of the series of obtained solution. In addition, it is difficult to describe the eigenvalues contained in the solutions by some general formula, because they are found from the solving power algebraic (characteristic) equations, which can be solved only by numerical or graphical methods [1, 2].

The application of classical analytical methods and methods of integral transformations together with variational methods and methods of weighted residuals (L. V. Kantorovich, Bubnov–Galyorkin orthogonal methods) allows to obtain solutions that do not contain the Bessel functions. However, for a large number of approximations, the accuracy of determining the eigenvalues is insufficient, which practically excludes these methods in obtaining solutions for small and ultra-small values of the time variable [3 ,4].

For obtaining analytic solutions to boundary-value problems for both plane solids and solids with central symmetry, the integral method of heat balance [5, 6] is very effective and promising. Characterized by its great universality and simplicity, this method also has significant drawbacks due to the low accuracy of the obtained solutions. In papers [7–9], in order to improve the solution accuracy, additional boundary conditions are introduced, which are defined so that their fulfillment by the desired solution is equivalent to the fulfillment of the equation of the boundary-value problem at the boundary points. It is shown that the fulfillment of the equation at the boundary points with increasing number of

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