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ON THE ESTIMATION OF THE CONVERGENCE RATE IN THE MULTIDIMENTIONAL LIMIT THEOREM FOR THE SUM OF WEAKLY DEPENDENT RANDOM VARIABLES FUNCTIONS

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Abstract

A refinement of estimates of the convergence rate obtained earlier in the multidimensional central limit theorem for the sums of vectors generated by the sequences of random variables with mixing is close to optimal. This has been achieved by imposing an additional condition on the characteristic functions of these sums, more accurate estimates of the semi-invariants, and using asymptotic expansions for the characteristic functions of the sums of independent random vectors. The result has been obtained using the summation methods for weakly dependent random variables based on S.N. Bernstein's idea of partition of the sums of weakly dependent random variables into long and short partial sums, as a result of which the long sums are almost independent, and the contribution of short sums to the total distribution is small. To estimate the differences between the sum distributions, we have used the S.M. Sadikova's inequality connecting the difference between the characteristic functions of random vectors with the difference between the corresponding distributions. To estimate the contribution of short sums, Markov and Bernstein's inequalities have been used.

Keywords: limit theorem, strong mixing, semi-invariants, asymptotic expansion, convergence rate

Random fields arise in the description of continuum mechanics processes for constructing physical relationships between various characteristics of the processes [1–5], plasma physics processes in calculating the velocity of plasma-chemical reactions [6–8], etc. In the statistical simulation of phenomena, limit theorems underlie the probabilistic criteria. The present paper is devoted to the use of asymptotic expansions of characteristic functions and estimates of the semi-invariants of sums of random variables to obtain estimates of the convergence rate in the limit theorem for the vector sequences generated by the sequences of random variables with mixing. We note that in [9] a new condition for the weak dependence of a sequence of random variables was introduced. In this case, the estimate of the rate of convergence in the central limit theorem for weakly dependent quantities is the same as for independent random variables. In [10, 11], the one-dimensional limit theorems (central and with large deviation) were proved and almost optimal estimates of the convergence rate were obtained. In [12–14], the multidimensional limit theorems for endomorphisms of Euclidean space were proved. In [15], the question of large deviations in the multidimensional limit theorem for trajectories generated by endomorphisms of the Euclidean space was investigated.

Let $a_1, a_2, \ldots$ be stationary sequence of random variables satisfying the strong mixing condition (introduced by M. Rozenblatt) in the narrow sense, with the coefficient

$\alpha(k) \le c\exp(-ak)$, where $c > 0$, $a > 0$ are the constants. We define random variables $\xi_{ij} = f_i(a_j, a_{j+1}, \ldots)$, $\xi_{ij}^s = E\{\xi_{ij}|a_j, \ldots, a_{j+s-1}\}$, where f_i are the measurable space maps of numerical sequences into a number line, $E\{\xi_{ij}|H\}$ is a conditional mathematical expectation with respect to a set of quantities H, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots$, $s = 1, 2, \ldots$

We form vectors $\xi_j = (\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj})$, $\xi_j^s = (\xi_{1j}^s, \xi_{2j}^s, \ldots, \xi_{mj}^s)$. We denote by P_n a distribution of the sum $\sum\limits_{j=1}^{n} \xi_j/\sqrt{n}$, $\Phi_R - m$ is the dimensional normal distribution with covariance matrix R and the zero vector of mathematical expectations. Besides, ω denotes an arbitrarily large fixed positive number. We assume that $s = s(n)$, $1 \le s(n) \le \omega \ln n$.

We consider the following conditions:

1) $E\xi_{i1} = 0$, $|\xi_{i1}| \le B$, where $|\cdot|$ denotes the vector length, $B > 0$ is a constant;

2) the matrix R with elements $r_{ij} = \lim\limits_{n\to\infty} E\Big(\sum\limits_{k=1}^{n} \xi_{ik}\Big)\Big(\sum\limits_{k=1}^{n} \xi_{jk}\Big)/n$ is non-degenerate;

3) $E|\xi_{i1} - E\{\xi_{i1}a_1, \ldots, a_r\}|^2 \le D\exp(-dr)$, where $d > 0$ is a constant;

4) $\overline{\lim\limits_{|t|\to\infty}} \sup\limits_{n} \Big|E\exp\Big(i\Big(t, \sum\limits_{j=1}^{n} \xi_j\Big)/\sqrt{n}\Big)\Big| < 1$, where $(t, \xi_j) = \sum\limits_{i=1}^{m} t_i\xi_{ij}$, $t = (t_1, t_2, \ldots, t_m)$.

Obviously, these conditions will be valid for ξ_{ij}^s.

Theorem 1. *Let us suppose that conditions* 1)–4) *are satisfied. Then,*

$$\sup_M |P_n(M) - \Phi_R(M)| = O(\ln^{m+4} n/\sqrt{n}),$$

where M are the convex measurable sets from an m-dimensional Euclidean space [1].

Proof. Let us prove the theorem for the sequence ξ_j^s. We present the estimates that are used in proving of the theorem.

Let us $a_{ij}^s = E\Big(\sum\limits_{k=1}^{l} \xi_{ik}^s\Big)\Big(\sum\limits_{k=1}^{l} \xi_{jk}^s\Big)/l.$

Lemma 1. *If $s = \omega \ln n$, then $a_{ij}^s = r_{ij} + O(1/l)$.*

Proof. First of all, let us consider how $E(\xi_{i1}\xi_{j,r+1})$ and $E(\xi_{i1}^s\xi_{j,r+1}^s)$ behave as r increases. We write

$$E(\xi_{i1}\xi_{j,r+1}) = E\{(\xi_{i1} - \xi_{i1}^{[r/2]+1})\xi_{j,r+1}\} + E(\xi_{i1}^{[r/2]+1}\xi_{j,r+1}).$$

Applying Holder's inequality and the strong mixing property, we obtain that

$$|E(\xi_{i1}\xi_{j,r+1})| \le E^{1/2}|\xi_{j,r+1}|E^{1/2}|\xi_{i,1} - \xi_{i,1}^{[r/2]+1}|^2 + 4B^2\alpha([r/2]),$$

where B is a constant.

Now, from the conditions of the theorems, we obtain that $|E\xi_{i1}\xi_{j,r+1}| \le C_1 r_{-\omega}$. Here and below, $C_i > 0$ are the constants. It is clear that this inequality is also satisfied for $|E\xi_{i1}^s\xi_{j,r+1}^s|$.

[1]Theorem 1 for $m = 1$ was proved in [16]. The asymptotic equality from theorem 1 will also hold for the stationary sequence ξ_j^s.

Through the stationarity of the sequences ξ_j^s, ξ_j, we have

$$a_{ij}^s = E\xi_{i1}^s\xi_{j1}^s + \sum_{r=1}^{l}(1-r/l)E\{\xi_{i1}^s\xi_{j,r+1}^s + \xi_{j1}^s\xi_{i,r+1}^s\} < \infty,$$

$$r_{ij} = E\xi_{i1}\xi_{j1} + \sum_{r=1}^{\infty}E\{\xi_{i1}\xi_{j,r+1} + \xi_{j1}\xi_{i,r+1}\} < \infty.$$

Hence,

$$|a_{ij}^s - r_{ij}| \le \sum_{r=0}^{l-1}E|\xi_{i1}\xi_{j,r+1} - \xi_{j1}^s\xi_{i,r+1}^s| + \sum_{r=1}^{l-1}E|\xi_{i1}\xi_{j,r+1} - \xi_{j1}^s\xi_{i,r+1}^s| +$$

$$+2\sum_{r=l+1}^{\infty}E|\xi_{i1}\xi_{j,r+1} + \xi_{j1}\xi_{i,r+1}| + \sum_{r=1}^{l-1}\frac{r}{l}|E\{\xi_{i1}^s\xi_{j,r+1}^s + \xi_{j1}^s\xi_{i,r+1}^s\}| = O(1/l),$$

Using Minkowski and Holder's inequalities, we have

$$\sum_{r=0}^{l-1}E|\xi_{i1}\xi_{j,r+1} - \xi_{j1}^s\xi_{i,r+1}^s| \le C_2 l(E\xi_{i1}^2 E|\xi_{i1} - \xi_{i1}^s|^2)^{1/2} \le C_2 l^{-\omega^{3/4}}.$$

The lemma is proved. □

We introduce the random variable $\tau_j = (t/|t|,\ \xi_j^s)$. Let $\chi_\nu(n)$ be a semi-invariant of the ν-order of the sum $\sum_{j=1}^{n}\tau_j$, i.e, $\chi_\nu(n) = \frac{d^\nu}{dz^\nu}\ln E\exp\Big(z\sum_{j=1}^{n}\tau_j\Big)\Big|_{z=0}$.

Lemma 2. *There is a constant H independent of ν, such that the estimate*

$$|\chi_\nu(n)| < H^\nu(\nu!)^2 n\, s^{\nu-1},\ 1 \le s \le \omega\ln n$$

holds.

The proof of the lemma is carried out in the same way as in [16].

Lemma 3. *If $\nu \le C_2\sqrt{k/\ln k}$, then* $E\Big|\sum_{j=1}^{k}\xi_j^s\Big|^{2\nu} \le C_3^{2\nu}(\ln k)^\nu k^\nu(2\nu)!$.

The lemma is proved similarly to lemma 2 in [17].

In the proof of the theorem, we shall use the summation method for weakly dependent random variables. It is based on breaking up the sum of weakly dependent random variables into long and short partial sums. As a result, long sums are almost independent, and the contribution of short sums to the total distribution is small. Later, instead of ξ_{ij}^s, we use ξ_{ij} and, without any loss of generality, assume that the matrix R^s is a unit matrix.

The sum, the distribution of which we study, is $S_n = \sum_{j=1}^{n}\xi_j$.

Let Q and N be natural numbers that grow together with n and satisfy the condition $|n - p(Q+N)| \le p$.

The sum S_n is divided as follows

$$S_n = \sqrt{Q}\sum_{j=1}^{p}y_j + \sqrt{Q}\sum_{j=1}^{p+1}y_j^0 = \sqrt{Q}(z_p + z_p^0), \quad y_j = (1/\sqrt{Q})\sum_{r=1}^{Q}\xi_{(j-1)(Q+N)+r},$$

$$y_j^0 = (1/\sqrt{Q})\sum_{r=1}^{N} \xi_{jQ+(j-1)N+r}, \quad y_{p+1}^0 = \sum_{r=1}^{n} \xi_{p(Q+N)+r}.$$

We denote by $\hat{y}_j$, $j = 1, 2, \ldots, p$, the independent random vectors distributed in the same way as y_1 and let $\hat{z}_p = \sum_{j=1}^{p} \hat{y}_j$. Let Λ be the covariance matrix of the vector y_1. By lemma 1, the elements of Λ differ from the elements of the identity matrix by the value $O(1/Q)$. We denote by A a matrix, such that $A'A = \Lambda^{-1}$, where A' is the transpose of the matrix A. Obviously, the vector $A\hat{z}_p/\sqrt{p}$ has a unitary covariance matrix.

Let G_p be the distribution of $z_p/\sqrt{p}$, G_p^A be the distribution $Az_p/\sqrt{p}$, $f_p(t)$, $\widehat{f}_p(t)$ be the characteristic functions $Az_p/\sqrt{p}$, $A\widehat{z}_p/\sqrt{p}$, respectively.

If $N = 2[\omega_1 \ln n]$, $1 \le s \le \omega_1 \ln n$, then

$$|f_p(t) - \widehat{f}_p(t)| \le C_4\, p/n^{\omega}. \tag{1}$$

This inequality follows from the relations (7), (8) of [16].

We denote

$$g_{\nu p}(t) = \exp(-|t^2|/2)\Big(1 + \sum_{r=1}^{\nu} P_r(it)(1/\sqrt{p})^r\Big),$$

where $P_r(it)$ depends on the semi-invariants of (Ay_1, it) of order at most $r+2$:

$$P_r(it) = \chi_{r+2}(it)/(r+2) + \sum_{l=1}^{r-1}\sum_{j_l=l}^{r-1}\sum_{j_{l-1}=l-1}^{j_l-1}\ldots\sum_{j_2=2}^{j_3-1}\times$$
$$\times \sum_{j_1=1}^{j_2-1} \frac{(r-j_1)(j_l-j_{l-1})\ldots(j_2-j_1)\chi_{r-j_l+2}(it)\chi_{j_l-j_{l-1}+2}\ldots\chi_{j_1+2}(it)}{(r-j_l+2)!(j_l-j_{l-1}+2)!\ldots(j_2-j_1+2)!(j_1+2)!}.$$

For the characteristic function of the sum $A\hat{z}_p/\sqrt{p}$, the following asymptotic expansion holds: for $|t| \le \sqrt{p}/(8(E|Ay_1|^{\nu+3})^{1(\nu+1)}) = T_{\nu p}$ takes place

$$\hat{f}_p(t) = g_{\nu p}(t) + O(3^{\nu+2}|t|^{\nu+3}\exp(-|t|^2/4)/T_{\nu p}^{\nu+1}), \tag{2}$$

(see [18], relations (7) and (8)). It follows from lemma 2 that

$$\begin{aligned} |\chi_r(it)| &= O((r!)^2 H^r s^{r-1}|t|^r/Q^{(r-2)/2}),\\ |P_r(it)| &= O\Big(\sum_{l=1}^{r} H^{r+2l} r^{2r+2l}|t|^{r+2l}s^{r+2l-1}/Q^{(l+r-1)/2}\Big). \end{aligned} \tag{3}$$

We use Sadikova's inequality [19], which relates the difference between the characteristic functions with the difference between the corresponding distributions

$$|G_p^A(M) - \Phi(M)| \le c_m r^{(m-1)/2}[I_1 + 2\sqrt{2}I_2 + 4\sqrt{2}I_3/T] +$$
$$+ 3\omega_\eta(\delta) + 2P\{|\eta| \ge r\} + 4P\{|\varsigma| > \delta\},$$

where η is the normal random vector with distribution Φ, ς is a vector with characteristic function $g(t) = \exp(-\sigma^2|t|^2/2)$ and density $(2\pi\sigma^2)^{-m/2}\exp(-|x|^2/2\sigma^2)$,

$$I_1^2 = \int_{|t|\le 1} |t|^{-2}|f_p(t) - \exp(-|t|^2/2)|\, dt, \quad I_2^2 = \int_{1<|t|\le T} |f_p(t) - \exp(-|t|^2/2)|\, dt,$$

$$I_3^2 = \int\limits_{|t|>T} |h(t)|^2 dt, \quad c_m = (2\pi)^{-m} \sqrt{\lambda_m S(O_1)}.$$

Here, O_1 is the ball of unit radius with the center at the origin, $S(O_1)$ is the surface of the sphere, $\lambda_k = (2\pi)^{m+1} \left(s \int\limits_0^\pi \sin^m \alpha d\alpha \right)^{-1}$, $\omega_\eta(\delta)$ is the least upper bound over all convex sets M of the probability of hit of a random vector η in the δ-neighborhood of a boundary M.

The technique of using Sadikova's inequality is described in [18].In our case, it is necessary to choose

$$T = C_4 \sqrt{Q}\, T_{\nu p}, \quad r = \sqrt{\frac{2\pi}{e}\, m \ln T}, \quad \sigma = \sqrt{m(m+1)\ln T}/T, \quad \delta = \sigma r.$$

From lemma 3, we obtain that

$$\begin{gathered} m^{(\nu+3)/2} \le E|Ay_1| \le C_5^{\nu+3}(\nu+3)!(\ln Q)^{(\nu+3)/2}, \\ C_6 \frac{\sqrt{p}}{\nu \ln Q} < T_{\nu p} < C_7\sqrt{p}, \quad C_7 \frac{\sqrt{p\,Q}}{\nu \ln Q} < T < C_9 \sqrt{p\,Q}. \end{gathered} \tag{4}$$

In the same way as in [20], we obtain

$$\begin{gathered} I_3 = O(1/T), \quad c_m r^{(m-1)/2} = O((\ln T)^{(m-1)/4}), \\ \omega_\eta(\delta) = O(\ln T/T)), \quad P\{|\eta| \ge r\} = P\{|\varsigma| \ge \sigma\} = O(1/T). \end{gathered} \tag{5}$$

The integrand in I_1 and I_2 is estimated as follows

$$\begin{aligned} |f_p(t) - \exp(-|t|^2/2)| \le |f_p(t) - \widehat{f}_p(t)| + |\widehat{f}_p(t) - \\ - g_{\nu p}(t)| + |g_{\nu p}(t) - \exp(-|t|^2/2)| = F_1(t) + F_2(t) + f_3(t). \end{aligned}$$

The integrals I_1 and I_2 are estimated by the sums of three integrals

$$\begin{aligned} I_1 &\le \sum_{i=1}^3 \left(\int\limits_{|t|\le 1} F_i^2(t)/|t|^2\, dt \right)^{1/2} = \sum_{i=1}^3 I_1^{(i)}, \\ I_2 &\le \sum_{i=1}^3 \left(\int\limits_{1\le|t|\le T} F_i^2(t)/|t|^2\, dt \right)^{1/2} = \sum_{i=1}^3 I_2^{(i)}. \end{aligned}$$

Using (1), we get $I_1^{(1)} = O(n^{-\omega}\sqrt{p/Q})$. According to relations (2) and (4),

$$I_1^{(2)} = O(3^{\nu+2}/T_{\nu p}^{\nu+1}) = O(3^{\nu+2}\nu^\nu \ln^\nu Q/p^{(\nu+1)/2}).$$

From relations (3), we obtain

$$I_1^{(3)} = O\Big(\Big(\int\limits_{|t|\le 1} \Big(\sum_{r=1}^{\nu} H^r s^r r^{2r}/(p\,Q)^{r/2} \Big)^2 dt \Big)^{1/2} \Big) = O(1/\sqrt{p\,Q}).$$

Thus, we estimated I_1.

Similarly to the estimates of $I_1^{(1)}$ and $I_1^{(3)}$, we obtain

$$I_2^{(1)} = O(pT^m\sqrt{p/Q}/n^\omega),\ I_2^{(3)} = O(1/\sqrt{p\,Q}).$$

The estimate of the integral $I_2^{(2)}$ is somewhat more complicated. We divide the domain of its integration into two sub-domains: $1 \le |t| \le T_{\nu p}$ and $T_{\nu p} < |t| < T$. By virtue of relation (2), the integral $I_2^{(21)}$ of the first sub-domain is estimated as

$$O(C_9^{\nu+2}/T_{\nu p}^{\nu+1}) = O(C_{10}^{\nu+2}\nu!\ln^\nu Q/p^{(\nu+1)/2}).$$

The integral $I_2^{(22)}$ of the second sub-domain is also divided into two integrals and we estimate each of them separately

$$I_2^{(22)} = \left(\int\limits_{T_{\nu p}<|t|\le T}|\widehat{f}_p(t)|^2dt\right)^{1/2} + \left(\int\limits_{T_{\nu p}<|t|\le T}|g_{\nu p}(t)|dt\right)^{1/2} =$$

$$= \left(\int\limits_{T_{\nu p}/\sqrt{p}<|t|\le T/\sqrt{p}} p^m|f(At)|^{2p}dt\right)^{1/2} + O(\exp(-T_{\nu p}/4)).$$

According to condition 4) of the theorem, there exists a positive constant α, such that $|f(At)| \le \exp(-\alpha)$. Therefore, we get that

$$I_2^{(22)} = O((pQ)^{m/2}\exp(-\alpha p) + \exp(-C_11\sqrt{p}/\ln^\nu Q)).$$

The obtained estimates make it possible to write down that

$$\sup|G_p^A(M) - \Phi(M)| = O(\ln^{m/4}(p\,Q)((p\,Q)^{(m+1)/2}/n^\omega +$$
$$+ p^{(m+3)/2}Q^{(m-1)/2}/n^\omega + C_{11}^{\nu+2}\nu^\nu\ln^\nu Q/p^{(\nu+1)/2} +$$
$$+ (p\,Q)^{m/2}\exp(-\alpha p) + \exp(C_{11}\sqrt{p}/\nu/\ln Q) + 1/\sqrt{pQ}) + \nu\ln^2(p\,Q)/\sqrt{p\,Q}). \quad (6)$$

Since the distance $\sup\limits_M|G_p^A(M) - \Phi(M)|$ is invariant under non-degenerate linear transformations, then the same estimate holds for $\sup\limits_M|G_p(M) - \Phi_\Lambda(M)|$. The elements of the matrix Λ differ from the elements of the identity matrix by $O(1/Q)$. Therefore, the estimate $\sup\limits_M|G_p(M) - \Phi(M)|$ differs from the previous estimate of (6) by $O(1/Q)$. Following this, we proceed from the estimate of the distribution $G_p(M) = P(z_p/\sqrt{p} \in M)$ to the estimate of the distribution $P((z_p + z_p^0)/\sqrt{p} \in M) = P(1/\sqrt{p\,Q}\sum\limits_{j=1}^n\xi_j \in M)$ in the same way as in [18, p. 94], but using lemma 3. We obtain a difference of order $O(\sqrt{N\ln N/Q} + 1/n^\omega)$. We proceed similarly to the estimate of the distribution $P(1/\sqrt{n}\sum\limits_{j=1}^n\xi_j \in M)$ and obtain the difference of order $O((N+1)/\sqrt{Q} + c_7^{2\nu}\ln^\nu n(2\nu)!n^\nu/(Q^{2\nu}p^\nu))$. Choosing $p = O(\omega_5\ln^5 n)$, $\nu = \ln n$, $Q = O(n/\ln^5 n)$ we obtain a statement of the theorem for the sequence ξ_j^s.

To complete the proof, we need to show that the error from the replacement of ξ_j^s by ξ_j goes to the remainder term.

From Minkowski's inequality and condition 3) of the theorem it follows that $E|\sum\limits_{j=1}^n\xi_{ij} - \sum\limits_{j=1}^n\xi_{ij}^s|^2 = O(1/n^\omega)$. Then, using the Markov's inequality operating in the same way as in [18], we obtain the assertion of the theorem. □

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Об оценке скорости сходимости в многомерной предельной теореме для сумм функций от слабо зависимых случайных величин

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Аннотация

Проведено близкое к оптимальному уточнение полученных ранее оценок скорости сходимости в многомерной центральной предельной теореме для сумм векторов, порожденных последовательностями случайных величин с перемешиванием. Этого удалось достичь за счет наложения дополнительного условия на характеристические функции этих сумм, более точных оценок их семиинвариантов и использования асимптотических разложений для характеристических функций сумм независимых случайных векторов. Результат получен с использованием методов суммирования слабо зависимых случайных величин, основанных на идее С.Н. Бернштейна разбивать суммы слабо зависимых случайных величин на длинные и короткие частичные суммы, в результате чего длинные суммы почти независимы, а вклад коротких сумм в общее распределение мал. Для оценки разностей между распределениями сумм используется неравенство С.М. Садиковой, связывающее разность между характеристическими функциями случайных векторов с разностью между соответствующими распределениями, а для оценки вклад коротких сумм -неравенства Маркова и Бернштейна.

Ключевые слова: предельная теорема, сильное перемешивание, семиинварианты, асимптотическое разложение,скорость сходимости

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