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OBSERVER'S MATHEMATICS APPLICATIONS TO NUMBER THEORY, GEOMETRY, ANALYSIS, CLASSICAL AND QUANTUM MECHANICS

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Abstract

When we consider and analyze physical events with the purpose of creating corresponding models, we often assume that the mathematical apparatus used in modeling is infallible. In particular, this relates to the use of infinity in various aspects and the use of Newton's definition of a limit in analysis. We believe that is where the main problem lies in contemporary study of nature. This work considers mathematical and physical aspects in a setting of arithmetic, algebra, geometry, and topology provided by Observer's Mathematics, see `www.mathrelativity.com`.

Key words: Hilbert, soliton, wave, Schrödinger, Lorentz, Schwartz, observer.

Introduction

Today, when we see the classical definition of a limit of a sequence (sequence a_n approaches a limit b if for any arbitrarily small number $\epsilon > 0$ there is an integer N, such that $|a_n - b| < \epsilon$ for all $n > N$), we feel somewhat uneasy: what does "arbitrarily small" really mean? Also, what does "sufficiently large" mean? This is because the answer depends on the point of view, depends on an observer, i.e., has relativistic characteristics.

Consider, for example, geometry. When we speak about lines, planes, or geometrical bodies, we understand that all these objects exist only in our imagination: even if we grind a metal plate we would never get an ideal plane because of instrument and operation. Moreover, we would never reach an ideal plane shape because of the atomic structure of the metal, i.e., we are not able to approach this shape with an arbitrary accuracy. In order to avoid the use of infinity, David Hilbert had created geometrical bases practically without the use of continuity axioms: Archimedes and completeness.

We find similar problems occurring in arithmetic, and in entire mathematics, since it is "arithmetical" in nature.

Physics encounters such problems as well. It is known fact that the dynamics of some systems change when we change the scale (distances, energies) at which we probe it. For example, consider a fluid. At each distance scale, we need a different theory to describe its behavior:

1. *At* ~ 1 cm – classical continuum mechanics (Navier – Stokes equations);
2. *At* $\sim 10^{-5}$ cm – theory of granular structures;
3. *At* $\sim 10^{-8}$ cm – theory of atom (nucleus + electronic cloud);
4. *At* $\sim 10^{-13}$ cm – nuclear physics (nucleons);
5. *At* $\sim 10^{-13} - 10^{-18}$ cm – quantum chromodynamics (quarks);
6. *At* $\sim 10^{-33}$ cm – string theory.

The mathematical apparatus that is applied here for physical data processing and building mathematical models does not contain any barriers, it is universal, omnivorous, and can manipulate with any numbers. This creates a possibility to produce an incorrect output. Observer's mathematics was created as an attempt to do away with the concept of infinity.

Proof of all theorems stated below can be found in [1–9].

1. Observer's mathematics applications to number theory

1.1. Analogy of Fermat's last problem. This result was presented by authors at the International Congress of Mathematicians in Madrid in 2006.

To begin, we present a few notes. It is obvious that the classical Fermat's Last problem (for any integer m, $m \geq 3$, there do not exist positive integers a, b, c, such that $a^m + b^m = c^m$) may be reformulated not just for integers a, b, c, but for any real rational numbers a, b, c.

Note, in observer's mathematics the power operation is not always associative. For illustrative purposes, we give a W_2 example. Consider $1.49 \in W_2$. Then $1.49 \times_2 1.49 = 2.14$ and $1.49 \times_2 2.14 = 3.16$. On the other hand, $1.49 \times_2 3.16 = 4.67$ and $2.14 \times_2 2.14 = 4.57$, i.e., $((1.49 \times_2 1.49) \times_2 1.49) \times_2 1.49 \neq (1.49 \times_2 1.49) \times_2 (1.49 \times_2 1.49)$.

Theorem 1. *For any integer n, $n \geq 2$, and for any integer m, $m \geq 3$, $m \in W_n$ there exist positive a, b, $c \in W_n$, such that $a^m +_n b^m = c^m$. Here x^m means $\underbrace{((\ldots (x \times_n x) \times_n \ldots) \times_n x))}_{m}$.*

For example, if $n = 2$, we can calculate that $1^3 +_2 1^3 = 1.28^3$.

Note that the main reason of cardinal difference between standard mathematics and observer's mathematics results is the following. The negative solution of classical Fermat's problem requires the Axiom of Choice to be valid. But in observer's mathematics this axiom is invalid.

1.2. Analogy of Mersenne's and Fermat's numbers problems. Mersenne's numbers are defined as $M_k = 2^k - 1$, with $k = 1, 2, \ldots$ The following question is still open: is every Mersenne's number square-free?

Fermat's numbers are defined as $F_k = 2^{2^k} + 1$, $k = 0, 1, 2, \ldots$ The following question is still open: is every Fermat's number square-free?

We begin with some comments. It is obvious that if some integer number is square-free in the set of all real integers, then this number is square-free in the set of all real rational numbers.

Theorem 2. *There exist integers n, $k \geq 2$, Mersenne's numbers M_k, with $\{k, M_k\} \in W_n$, and positive $a \in W_n$, such that $M_k = a^2$.*

Theorem 3. *There exist integers n, $k \geq 2$, Fermat's numbers F_k, $\{k, F_k\} \in W_n$, and positive $a \in W_n$, such that $F_k = a^2$.*

1.3. Analogy of Waring's problem. It is known (Lagrange) that the minimum number of squares to express all positive integers is four. What is the minimum number of k-th powers necessary to express all positive integers? This is a classical Waring's problem in standard arithmetic.

Theorem 4. *For any integer k, $k \geq 2$, there exist integer n, $n \geq 2$, $(k \in W_n)$ and some $x \in W_n$, such that any equality of the form $x = a_1^k + a_2^k + \ldots + a_l^k$ is not possible for any integer $l \in W_n$ and any positive numbers a_1, a_2, $\ldots$, $a_l \in W_n$.*

Note that for $n = 2$ and for any $x \in W_2$, $x \in [0, 1]$, there do not exist more than four numbers a, b, c, $d \in W_2$, such that $x = ((a^2 +_2 b^2) +_2 c^2) +_2 d^2$.

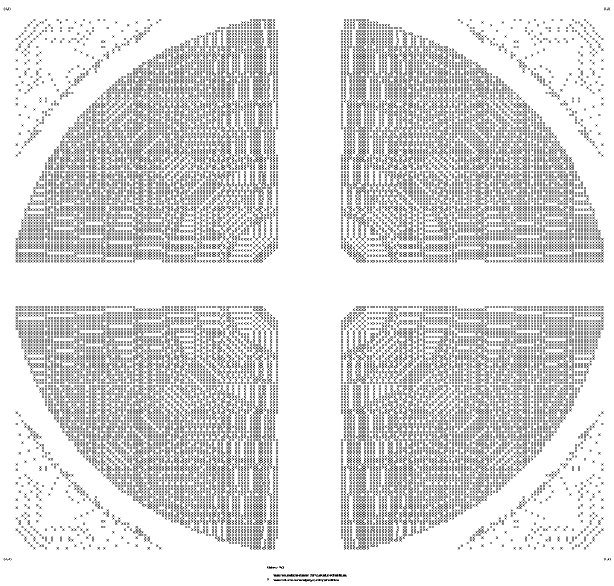

Fig. 1. Nadezhda effect

1.4. Tenth Hilbert problem in observer's mathematics. We provide the following

Theorem 5. *For any positive integers* $m, n, k \in W_n$, $n \in W_m$, $m > log_{10}(1 + (2 \cdot 10^{2n} - 1)^k)$, *from the point of view of the* W_m-*observer, there is an algorithm that takes as input a multivariable polynomial* $f(x_1, \ldots, x_k)$ *of degree* q *in* W_n *and outputs YES or NO according to whether there exist* $a_1, \ldots, a_k \in W_n$, *such that* $f(a_1, \ldots, a_k) = 0$.

Therefore, Hilbert's tenth problem in observer's mathematics has positive solution. We think that Hilbert expected a positive answer for his tenth problem. Note that the main reason of cardinal difference between standard mathematics and observer's mathematics results is the following. The negative solution of the classical tenth problem requires the Axiom of Choice to be valid. But in observer's mathematics this Axiom is invalid.

2. Observer's Mathematics application to geometry: Nadezhda effect

In this section we consider an open square Q centered at the origin with sides of length 2 located on a plane $W_n \times W_n$. We will calculate the distance D between the origin $(0,0)$ and any point of Q as follows. $D = \rho((0,0),(x,y)) = \sqrt{x^2 + y^2} = \sqrt{x \times_n x +_n y \times_n y}$, where $\sqrt{a} = b$ means $b \times_n b = a$, $x, y \in Q$, i.e., $|x| < 1$, $|y| < 1$.

Fig. 1 below contains an illustration of the fact that for some points on $W_n \times W_n$ the concept of distance from the origin does not exist, while for others it does exist. The illustration below is for $n = 3$ ($Q \subset W_3 \times W_3$). Points with no distance to the origin are indicated by black, while points where distance from the origin exists are indicated in white.

This means that the distance D does not always exist, i.e., not every segment on a plane has a length. This phenomenon occurs for all n. We call the presence of these "black holes" and "white cross" as the Nadezhda effect (see Fig. 1). This effect gives us new possibilities for discovering physical processes and developing their mathematical models.

3. Observer's mathematics application to analysis and physics

In classical physics, it has been realized for centuries that the behavior of idealized vibrating media (such as waves on string, on a water surface, or in air), in the absence of friction or other dissipative forces, can be modeled by a number of partial differential equations known collectively as dispersive equations. Model examples of such equations include the following:

• The free wave equation $u_{tt} - c^2\Delta u = 0$ where $u : R \times R^d \to R$ represents the amplitude $u(t,x)$ of a wave at a point in spacetime with d spatial dimensions, $\Delta = \sum_{j=1}^{d} \frac{\delta^2}{\delta x_j^2}$ is the spatial Laplacian on R^d, u_{tt} is short for $\frac{\delta^2 u}{\delta t^2}$, and $c > 0$ is a fixed constant.

• The linear Schrödinger equation $i\hbar u_t + \frac{\hbar^2}{2m}\Delta u = Vu$ where $u : R \times R^d \to R$ is the wave function of a quantum particle, $\hbar, m > 0$ are physical constants and $V : R^d \to R$ is a potential function, which we assume to depend only on the spatial variable x.

The theory of linear dispersive equations predicts that waves should spread out and disperse over time. However, it is a remarkable phenomenon, observed both in theory and practice, that once nonlinear effects are taken into account, solitary wave and soliton solutions can be created, which can be stable enough to persist indefinitely.

From the point of view of W_n-observer (we will call such observers "naive", since they "think" that they "live" in W and deal with W) a real function y of a real variable x, $y = y(x)$, is called differentiable at $x = x_0$ if there is a derivative

$$y'(x_0) = \lim_{x \to x_0,\, x \neq x_0} \frac{y(x) - y(x_0)}{x - x_0}.$$

What does the above statement mean from the point of view of W_m-observer with $m > n$? It means that

$$|(y(x) -_n y(x_0)) -_n (y'(x_0) \times_n (x -_n x_0))| \leq 0.\underbrace{0\ldots01}_{n}$$

whenever

$$|y(x) -_n y(x_0)| = 0.\underbrace{0\ldots0y_l}_{l}\, y_{l+1}\ldots y_n$$

and

$$|(x -_n x_0)| = 0.\underbrace{0\ldots0x_k}_{k}\, x_{k+1}\ldots x_n$$

for $1 \leq k,\ l \leq n$, and x_k being non-zero digit. The following theorems have been proven:

Theorem 6. *From the point of view of a W_m-observer a derivative calculated by a W_n-observer $(m > n)$ is not defined uniquely.*

Theorem 7. *From the point of view of a W_m-observer (with $m > n$) $|y'(x_0)| \leq$ $\leq C_n^{l,k}$, where $C_n^{l,k} \in W_n$ is a constant defined only by n, l, k and not dependent on $y(x)$.*

Theorem 8. *From the point of view of a W_m-observer, when a W_n-observer (with $m > n \geq 3$) calculates the second derivative*

$$y''(x_0) = \lim_{x_1 \to x_0, x_1 \neq x_0, x_2 \to x_0, x_2 \neq x_0, x_3 \to x_1, x_3 \neq x_1} \frac{\dfrac{y(x_3) - y(x_1)}{(x_3 - x_1)} - \dfrac{y(x_2) - y(x_0)}{x_2 - x_0}}{x_1 - x_0},$$

we get the following unequality:

$$(|x_2 -_n x_0| \times_n |x_3 -_n x_1|) \times_n |x_1 -_n x_0| \geq 0.\underbrace{0\ldots01}_{n}$$

provided that $y''(x_0) \neq 0$.

3.1. Free wave equation. We consider the case when $d = 1$, i.e., $u : W_n \times W_n \to \to W_n$, from W_m-observer point of view, with $m > n$, where $W_n \times W_n$ means Cartesian product of W_n with itself. The free wave equation may be written as

$$u_{tt} -_n ((c \times_n c) \times_n u_{xx}) = 0.$$

Then we have the following

Theorem 9. *Let*

$$c = c_0.c_1 \ldots c_k c_{k+1} \ldots c_n$$

and

$$u_{xx} = \pm u_0^{xx}.u_1^{xx} \ldots u_l^{xx} u_{l+1}^{xx} \ldots u_n^{xx}$$

with $2k < n$, $l < n$, $c_0 = c_1 = \ldots = c_k = 0$, $c_{k+1} \neq 0$, $u_0^{xx} = u_1^{xx} = \ldots = u_l^{xx} = 0$ and $u < k + l + 2$, then $u_{tt} = 0$.

Next, we have the following

Theorem 10. *If $d_0 \geq \underbrace{9\ldots9}_{p}$, with $0 < p \leq n$ and $u_0^{xx} \geq \underbrace{9\ldots9}_{q}$, with $0 < q \leq n$ and $n < p + q$, then there is no u_{tt}, such that $u_{tt} = ((c \times_n c) \times_n u_{xx})$.*

3.2. Schrödinger equation. Consider the following:

$$-(\hbar \times_n \hbar) \times_n \Psi_{xx} +_n ((2 \times_n m) \times_n V) \times_n \Psi = i((2 \times_n m) \times_n \hbar)\Psi_t,$$

where $\Psi = \Psi(x, t)$, $\hbar$ is the Planck's constant, $\hbar = 1.054571628(53) \cdot 10^{-34}$ m^{2}kg/s. Then we have the following

Theorem 11. *Let $36 < n < 68$, $m = m_0.m_1 \ldots m_k m_{k+1} \ldots m_n$, with $m \in W_n$, $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$, $k + 35 < n$, $V = 0$, then $\Psi_t = \Psi_t^0.\Psi_t^1 \ldots \Psi_t^l \Psi_t^{l+1} \ldots \Psi_t^n$ and $\Psi_t^0 = \ldots \Psi_t^l = 0$, $\Psi_t^{l+1}, \ldots, \Psi_t^n$ are free and in $\{0, 1, \ldots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with $\Psi_t \in \in \{(0.\overbrace{\underbrace{0\ldots0}_{l} * \ldots *}^{n})\}$, where $* \in \{0, 1, \ldots, 9\}$.*

Corollary 1. *Let $36 < n < 68$, $m = m_0.m_1 \ldots m_k m_{k+1} \ldots m_n$, with $m \in W_n$, $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$. Also, let $V = v_0.v_1 \ldots v_s v_{s+1} \ldots v_n$, with $V \in W_n$, $v_0 = v_1 = \ldots = v_s = 0$, $v_{s+1} \neq 0$, with $\begin{cases} k + 35 < n \\ k + s + 2 > n \end{cases}$, then $\Psi_t = \Psi_t^0.\Psi_t^1 \ldots \Psi_t^l \Psi_t^{l+1} \ldots \Psi_t^n$ and $\Psi_t^0 = \ldots \Psi_t^l = 0$, $\Psi_t^{l+1}, \ldots, \Psi_t^n$ are free and in $\{0, 1, \ldots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with $\Psi_t \in \{(0.\overbrace{\underbrace{0\ldots0}_{l} * \ldots *}^{n})\}$, where $* \in \{0, 1, \ldots, 9\}$.*

3.3. Two-slit interference. Quantum mechanics treats the motion of an electron, neutron or atom by writing down the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\delta^2\Psi}{\delta x^2}+V\Psi=i\hbar\frac{\delta\Psi}{\delta t},$$

where m is the particle mass and V is the external potential acting on the particle. As these particles pass through the two slits of any of the experiments they are moving freely; we, therefore, set $V=0$ in the Schrödinger equation.

Now, consider the following:

$$-(\hbar\times_n\hbar)\times_n\Psi_{xx}+_n((2\times_n m)\times_n V)\times_n\Psi=i((2\times_n m)\times_n\hbar)\Psi_t,$$

where $\Psi=\Psi(x,t)$, $\hbar$ is the Planck's constant, $\hbar=1.054571628(53)\cdot 10^{-34}$ $\mathrm{m^2kg/s}$. Then we have the following

Theorem 12. *Let* $36<n<68$, $m=m_0.m_1\ldots m_km_{k+1}\ldots m_n$, *with* $m\in W_n$, $m_0=m_1=\ldots=m_k=0$, $m_{k+1}\neq 0$, $k+35<n$, $V=0$, *then* $\Psi_t=\Psi_t^0.\Psi_t^1\ldots\Psi_t^l\Psi_t^{l+1}\ldots\Psi_t^n$ *and* $\Psi_t^0=\ldots\Psi_t^l=0$, $\Psi_t^{l+1},\ldots,\Psi_t^n$ *are free and in* $\{0,1,\ldots,9\}$, *where* $l=n-k-36$, *i.e.,* Ψ_t *is a random variable, with* $\Psi_t\in\{(0.\overbrace{\underbrace{0\ldots0}_{l}*\ldots*}^{n})\}$, *where* $*\in\{0,1,\ldots,9\}$.

The wave at the point of combination will be the sum of those from each slit. If Ψ_1 is the wave from slit 1 and Ψ_2 is the wave from slit 2, then $\Psi=\Psi_1+\Psi_2$. The result gives the predicted interference pattern. Then by Theorem 1, we have

$$\Psi_{1t}=\Psi_{1t}^0.\Psi_{1t}^1\ldots\Psi_{1t}^l\Psi_{1t}^{l+1}\ldots\Psi_{1t}^n,$$

$$\Psi_{2t}=\Psi_{2t}^0.\Psi_{2t}^1\ldots\Psi_{2t}^l\Psi_{2t}^{l+1}\ldots\Psi_{2t}^n,$$

$$\Psi_{1t}^0=\ldots=\Psi_{1t}^l=0,$$

where $\Psi_{1t}^{l_1+1},\ldots,\Psi_{1t}^n$ are free and in $\{0,1,\ldots,9\}$, and

$$\Psi_{2t}^0=\ldots=\Psi_{2t}^l=0,$$

where $\Psi_{2t}^{l_2+1},\ldots,\Psi_{2t}^n$ are free and in $\{0,1,\ldots,9\}$ where $l=n-k-36$.

Now we have the following

Theorem 13. 1. *If* $\Psi_{1t}^{l+1}+\Psi_{2t}^{l+1}>9$, *then* $\Psi_1+\Psi_2$ *is not a wave.*
2. *If* $\Psi_{1t}^{l+1}+\Psi_{2t}^{l+1}<9$, *then* $\Psi_1+\Psi_2$ *is a wave.*
3. *If* $\Psi_{1t}^{l+1}+\Psi_{2t}^{l+1}=9$, *then* $\Psi_1+\Psi_2$ *may or may not be a wave.*

3.4. Lorentz transform. Let K and K' be two inertial coordinate systems with x-axis and x'-axis permanently coinciding. We consider only events which are localized on the $x(x')$-axes. Any such event is represented with respect to the coordinate system K by the abscissa x and the time t, and with respect to the system k' by the abscissa x' and the time t' when x and t are given. A light signal, which is proceeding along the positive $x-$axis, is transmitted according to the equation $x=c\times_n t$ or $x-_n c\times_n t=0$ Since the same light signal has to be transmitted relative to k' with the velocity c, the propagation relative to the system k' will be represented by the analogous equation

$$x'-_n c\times_n t'=0.$$

Those space-time points (events) which satisfy the first equation must also satisfy the second equation. Obviously there will be the case when the relation $\lambda_1 \times_n (x' -_n c \times_n t') = = \mu_1 \times_n (x -_n c \times_n t)$ is fulfilled in general, where $\lambda_1, \mu_1 \in W_n$, $|\lambda_1|, |\mu_1| \geq 1$ are constants; for, according to the last equation, the disappearance of $(x -_n c \times_n t)$ involves the disappearance of $(x' -_n c \times_n t')$.

Note that classical equation $x' - ct' = \lambda(x - ct)$ is not valid since if $\lambda < 1$, $x - ct = = 0.\underbrace{0\ldots0}_{n-1}1$, then $\lambda \times_n (x -_n c \times t) = x' -_n c \times_n t' = 0$. If we apply quite similar considerations to light rays which are being transmitted along the negative x-axis, we obtain the condition $\lambda_2 \times_n (x' +_n c \times_n t') = \mu_2 \times_n (x +_n c \times_n t)$ with $\lambda_2, \mu_2 \in W_n$, $|\lambda_2|, |\mu_2| \geq 1$.

3.5. Schwarzian derivative. The Schwarzian derivative $S(f(x))$ is defined as $S(f(x)) = \dfrac{f'''(x)}{f'(x)} - \dfrac{3}{2}\left(\dfrac{f''(x)}{f'(x)}\right)^2$ Here $f(x)$ is a function in one real variable and $f'(x)$, $f''(x)$, $f'''(x)$ are its derivatives. The Schwarzian derivative is ubiquitous and tends to appear in seemingly unrelated fields of mathematics including classical complex analysis, differential equations, and one-dimensional analysis, as well as more recently, Teichmüller Theory, integrable systems, and conformal field theory. For example, let's consider the Lorentz plane with the metric $g = dxdy$ and a curve $y = f(x)$. If $f'(x) > 0$, then its Lorentz curvature can be easily computed via $\rho(x) = f''(x)(f'(x))^{-3/2}$ and the Schwarzian enters the game when one computes $\rho' = \dfrac{S(f)}{\sqrt{f'}}$. Thus, informally speaking, the Schwarzian derivative is curvature.

Consider now the Schwarzian curvature from observer's mathematics point of view. Now we have the following

Theorem 14. *If $S(f(x))$ exists, then*
- *$S(f(x))$ is a random variable;*
- *$|S(f(x)| \leq 10^{l-k+1}$, where*

$$(2 \times_n (f'(x) \times_n f'(x))) = 0.\underbrace{0\ldots0a_l}_{l}\, a_{l+1}\ldots a_n$$

with $a_l \neq 0$ and

$$(2 \times_n (f'''(x) \times_n f'(x))) -_n (3 \times_n (f''(x) \times_n f''(x))) = \pm 0.\underbrace{0\ldots0b_k}_{k}\, b_{k+1}\ldots b_n$$

with $b_k \neq 0$ and $1 < l,\, k < n$.

Резюме

Б. Хоц, Д. Хоц. Применение математики наблюдателя к теории чисел, геометрии, анализу, классической и квантовой механике.

При рассмотрении и анализе физических событий с целью создания соответствующих моделей мы часто предполагаем, что математический аппарат, используемый в моделировании, непогрешим. В частности, это касается использования бесконечности в различных аспектах и применения ньютоновского определения предела в анализе. Мы считаем, что именно в этом заключается основная проблема в современном изучении природы. В настоящей работе рассматриваются математические и физические аспекты арифметики, алгебры, геометрии и топологии математики наблюдателя (см. `www.mathrelativity.com`).

Ключевые слова: Гильберт, солитон, волна, Шредингер, Лоренц, Шварц, наблюдатель.

References

1. *Khots B., Khots D.* Mathematics of Relativity. – 2004. – URL: www.mathrelativity.com.
2. *Khots B., Khots D.* An Introduction to Mathematics of Relativity // Lecture Notes in Theoretical and Mathematical Physics / Ed. A.V. Aminova. – Kazana: Kazan State University, 2006. – V. 7. – P. 269–306.
3. *Khots B., Khots D.* Observer's Mathematics – Mathematics of Relativity // Appl. Math. Comput. – 2007. – V. 187, No 1. – P. 228–238.
4. *Khots D., Khots B.* Quantum Theory and Observer's Mathematics // AIP Conf. Proc. – 2007. – V. 962. – P. 261–264.
5. *Khots B., Khots D.* Analogy of Fermat's last problem in Observer's Mathematics – Mathematics of Relativity // Talk at the International Congress of Mathematicians. – Madrid, Spain, 2006. – URL: http://icm2006.org/v_f/AbsDef/Shorts/abs_0358.pdf.
6. *Khots B., Khots D.* Analogy of Hilbert's tenth problem in Observer's Mathematics // Talk at the International Congress of Mathematicians. – Hyderabad, India, 2010. – URL: http://www.icm2010.in/wp-content/icmfiles/abstracts/Contributed-Abstracts-5July2010.pdf.
7. *Khots B., Khots D.* Non-Euclidean Geometry in Observer's Mathematics // Acta Physica Debrecina. – 2008. – T. XLII, – P. 112–119.
8. *Khots B., Khots D.* Quantum Theory from Observer's Mathematics point of view // AIP Conf. Proc. – 2010. – V. 1232. – P. 294–298.
9. *Khots B., Khots D.* Solitary Waves and Dispersive Equations from Observer's Mathematics point of view // Geometry "in large", topology and applications. – Kharkov, Ukraine, 2010. – P. 86–95.

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