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Metric, Banach, and Hilbert Spaces of ϕ_B-Distributions

V. S. Mokeichev[1*]

[1]*Kasan Federal University*
ul. Kremlyovskaya 18, Kasan, 420008 Russia

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Abstract—We introduce ϕ-distributions and prove that their set is a metric space. We also consider a Banach space and a Hilbert space of such distributions. The results are applied to differential equations with Laurent's type coefficients.

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In [1] they introduced the notion of ϕ-solution of some linear equations. Indeed, generalized function (Schwartz distribution) is called ϕ-solution to equation $Au = f$, where $\phi = \{\phi_p(x), p \in N\}$, if $u = \sum u_p\phi_p(x)$, and series $\sum A(u_p\phi_p(x))$ converges to $f(x)$ in some space. This is the Fourier method for finding solutions to linear equations. Later it was found that this notion is very convenient for finding the solutions to linear boundary problems for differential equations in partial derivatives with deviations of the arguments [2]. However, strict fixation of ϕ-solutions with the set of generalized functions lead to the fact that some mathematical models were insoluble; this is not the way in G. Hadamard opinion. The insolubility of mathematical model means that a moment leaves out of account or the definition of solution is unsuccessfully chosen.

Example 1. The mathematical model of forced 2π-periodic oscillations of string fastened at 0 and π, is considered to be correct and has the form $u_{tt}^{(2)} - c^2 \cdot u_{xx}^{(2)} = f(t, x)$, where $u, u_t^{(1)}$ are 2π-periodic, $u(t, 0) = u(t, \pi) = 0$.

If c be irrational number, then expected solution has the form

$$\sum f_{k_1,k_2}(-k_1^2 + c^2k_2^2)^{-1} \exp{(ik_1t)} \sin(k_2x),$$

where summation is realized over $k_1 = 0, \pm1, \pm2, \ldots,$ $k_2 = 1, 2, \ldots,$ i is imaginary unit and $f(t, x) = \sum f_{k_1,k_2} \exp{(ik_1t)} \sin(k_2x)$. However, the written out series is divergent even in the space of generalized periodic functions if c is Liouville number, and $f_{k_1,k_2} = (1 + k_1^2 + k_2^2)^{-m}$, where $m \gg 0$. This allows to call the problem insoluble what contradicts physics of studied process. The reason of insolubility is fixation of the notion of solution with a space of generalized functions (Schwartz distribution).

In detail Example 1 is considered in [3]. In [3–5] the notion of ϕ-solution is introduced without fixation to a space of generalized functions. For this they introduced the notion of ϕ-distribution (the the notion of ϕ-solution) and developed the solvability theory of linear boundary-value problem for differential equations in partial derivatives with deviations of the arguments. In [6] they investigated the space of ϕ-distributions in case when ϕ is a set of functions; correctly defined the notion of differentiability and integrability of ϕ-distributions and studied the decomposability problem of generalized function in series by specified system of functions.

We want to transfer the obtained results on the systems linear equations (not at all differential). This leads to introduction of the notion of ϕ_B-distribution, where B is Banach space, and to answer questions: Is the method of compressed maps and Schauder theory can be transferred to the equations considered in a space of ϕ_B-distributions?

*E-mail: Valery.Mokeychev@kpfu.ru.