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Nonlocal Problems for an Equation of Mixed Parabolic-Hyperbolic Type with Power Degeneration

S. N. Sidorov[1]*

[1] *Sterlitamak Branch of the Bashkir State University, pr. Lenin 37, Sterlitamak, 453103 Russia*

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Abstract—For mixed type equation in a rectangle, we study boundary-value problems with nonlocal boundary condition that connects the values of the sought-for solution and its derivative along the normal at upper and lower bases of the rectangle which belong to different types of studied equation. By the method of spectral analysis we establish criteria of uniqueness of solutions that are constructed in the form of the sum of the Fourier series. We establish the stability of solutions for a nonlocal condition.

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1. INTRODUCTION

Consider an equation of mixed parabolic-hyperbolic type

$$Lu \equiv \begin{cases} t^n u_{xx} - u_t - b^2 t^n u = 0, & t > 0; \\ (-t)^m u_{xx} - u_{tt} - b^2 (-t)^m u = 0, & t < 0, \end{cases} \tag{1}$$

in a rectangular domain $D = \{(x,t) \mid 0 < x < 1, -\alpha < t < \beta\}$, where $n > 0$, $m > 0$, $b \geq 0$, $\alpha > 0$, and $\beta > 0$ are given real values.

Problem 1. Find in the domain D a function $u(x,t)$, satisfying the following conditions:

$$u(x,t) \in C^1(\overline{D}) \cap C^2(D_-) \cap C_x^2(D_+), \tag{2}$$

$$Lu(x,t) \equiv 0, \ \ (x,t) \in D_- \cup D_+, \tag{3}$$

$$u(0,t) = u(1,t) = 0, \ \ -\alpha \leq t \leq \beta, \tag{4}$$

$$u(x,-\alpha) - u(x,\beta) = \varphi(x), \ \ 0 \leq x \leq 1, \tag{5}$$

where $\varphi(x)$ is a given sufficiently smooth function, $\varphi(0) = \varphi(1) = 0$, $D_- = D \cap \{t < 0\}$, $D_+ = D \cap \{t > 0\}$.

Problem 2. Find in the domain D a function $u(x,t)$, satisfying conditions (2)–(4) and the condition

$$u_t(x,-\alpha) - u_t(x,\beta) = \psi(x), \ \ 0 \leq x \leq 1, \tag{6}$$

where $\psi(x)$ is a given sufficiently smooth function, while $\psi(0) = \psi(1) = 0$.

One of the first investigations of the conjugacy problems, when a parabolic equation is given on one part of a domain, and a hyperbolic equation on the other, is I. M. Gel'fand's work [1]. He considers an example connected with the motion of a gas in a channel, surrounded by porous medium, while in the channel motion of a gas is described by wave equation, and by diffusion equation outside it. Ya. S. Uflyand [2] reduced the problem of propagation of electric oscillation in composite lines, when

* E-mail: stsid@mail.ru.