

# REFLECTION PRINCIPLES, ALGEBRAS AND PROGRESSIONS OF THEORIES

LEV D. BEKLEMISHEV

Reflection principles are axioms expressing that all sentences (of a given logical complexity) provable in a theory  $T$  are true. The simplest example of such an axiom is Gödel's formula expressing the consistency of  $T$ .

The idea of using reflection principles and their transfinite hierarchies to classify arithmetical sentences according to strength is due to A. Turing (1939). However, Turing also realized that there are serious difficulties associated with this approach, in particular, due to the lack of understanding how to distinguish 'canonical' from 'pathological' ordinal notation systems, now a well-known problem in proof theory.

The aim of the series of two talks is to outline the main ingredients of the approach to proof-theoretic analysis based on reflection algebras. From an abstract algebraic point of view, these structures are semilattices enriched by a family of monotone unary operators satisfying some specific sets of identities. The operators can be interpreted in the lattice of arithmetical theories as functions mapping a theory  $T$  to a theory axiomatized by a reflection principle for  $T$ .

Within this framework it is possible to define appropriate canonical ordinal notation systems and the associated transfinite hierarchies of reflection principles. Hence, it is also possible, to some extent, to push Turing's ideas on the classification of arithmetical sentences through.

For those cases where such algebras have been sufficiently understood, this allows one to obtain a more refined ordinal classification of theories than using more traditional proof-theoretic approaches. This has been initially done for Peano arithmetic and its fragments by the author. In a joint paper with F. Pakhomov this approach is now extended to theories of predicative strength, such as the second-order theories of iterated arithmetical comprehension.

STEKLOV MATHEMATICAL INSTITUTE, MOSCOW

*E-mail address:* lbekl@yandex.ru

*URL:* <http://www.mi-ras.ru/~bekl/>