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On Geodesic Curves on Quotient Manifold of Nondegenerate Affinor Fields

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Abstract—We consider the quotient manifold of the manifold of nondegenerate affinor fields on a compact manifold with respect to the action of the group of nowhere vanishing functions. This manifold is endowed with a structure of infinite-dimensional Lie group. On this Lie group, we construct an object of linear connection with respect to which all left-invariant vector fields are covariantly constant (the Cartan connection). We also find the geodesics of the Cartan connection.

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INTRODUCTION

Here we consider the manifold T_0 of nondegenerate valency $(1,1)$ tensors of finite smoothness class on a compact manifold M. This manifold T_0 is subject to action of a group of functions F_0 that do not vanish at any point of the manifold M. The quotient manifold T_0/F_0 by the action of this group forms an infinite-dimensional Lie group. On any Lie group, we can define the Cartan connection with respect to which all left-invariant vector fields are covariantly constant. Lie groups admit three Cartan connections: left, right, and symmetric [1].

In [2], an object of the left Cartan connection on the quotient manifold of non-degenerate affinor fields was constructed, and in [3] certain families of geodesic curves were found in special cases. This paper is a continuation and generalization of [2] and [3].

We consider two methods for finding general solutions to the geodesic curve equations for the Cartan connection on the factor manifold T_0/F_0. Each of the methods is unique in its solution in view of the linear connection structure specificity on an infinite-dimensional manifold, therefore here we present both solving methods. According to the first method, finding of geodesic curves is locally reduced to solution to the corresponding second-order differential equation. The equation obtained in this case is characterized by the special solution complexity. The essence of the second solving method lies in the fact that the geodesic curves passing through the unit I on the Lie group T_0/F_0 are one-parameter subgroups. One-parameter subgroups of the Lie group can be identified with the integral curves of left-invariant vector fields passing through the unit I ([1], pp. 99–100). Here $I \in T_0$ is the unit affinor field.

Recall only the most important definitions and the theorem used in the proofs of the statements in [2] and [3].

Definition 1 ([4], P. 64, 5.12.1). A set G is called a *Lie group of class* C^{infty}, if

1) G is an algebraic group,

2) G is a manifold of class C^{∞},

3) there exist operations of multiplication and inversion of the class C^{∞}.

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