ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 12, pp. 56–72. © Allerton Press, Inc., 2015.
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# On Tail Dependence for Grubbs' Copula-Function

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Received May 30, 2014

**Abstract**—We consider Grubbs' statistics for a normal sample. Those statistics are the standardized maximum and standardized minimum. We investigate the distribution properties of Grubbs' statistics and construct a new copula-function by an inversion method from the joint distribution of Grubbs' statistics. We also describe properties of the constructed Grubbs' copula-function. It is proved that this copula is symmetric. We give examples of plots of the simulated values from Grubbs' copula. It is found that Grubbs' copula-function allows to describe negative dependencies between random variables. It is proved that for Grubbs' copula-function coefficients of the upper-left and lower-right tail dependencies are equal each other. We find the formula for calculation of these coefficients and execute model calculations of tail dependencies coefficients for Grubbs' copula.

**DOI:** 10.3103/S1066369X15120063

Keywords: *copula-function, tail dependence, upper-left and lower-right tail dependencies coefficients, symmetric copula, joint distribution function of standardized maximum and minimum, normal distribution.*

## 1. INTRODUCTION

Coupla models find a wide range of applications in modeling of natural phenomena (e.g., meteorology and oceanology), as well as in problems of financial and insurance mathematics. This is explained by the fact that copulas allow modeling of multivariate non-Gaussian distributions of random variables, that can be observed in many subject areas [1]. Copula-function can be extracted from a known joint distribution of random variables. Such a way of construction of copula-function is based on a well-known Sklar's theorem [1]. The goal of this paper is the construction of a new copula-function by inversion from joint distribution of one-sided Grubbs' statistics and the study of its properties.

## 2. CONSTRUCTION OF GRUBBS' COPULA

Let $X_1, X_2, \ldots, X_{n-1}, X_n$ be a random sample of $n$ values of a normally distributed random variable $X$; $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n-1)} \le X_{(n)}$ be the ordered observations of this sample; $X_{(j)}$ be the $j$-th order statistic ($j = \overline{1,n}$).

For testing normal sample for outliers F. Grubbs proposed a range of special statistics. The examples of such characteristics are standardized maximum and minimum, calculated by a sample of $n$ observations [2]:

$$T_n^{(1)} = (X_{(n)} - \overline{X})/S, \tag{1}$$

$$T_{n,(1)} = (\overline{X} - X_{(1)})/S, \tag{2}$$

where $\overline{X} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1}\sum\limits_{i=1}^{n}(X_i - \overline{X})^2$.

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