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Topological Methods in One Numerical Scheme of Solving Three-Dimensional Continuum Mechanics Problems

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Abstract—We discuss numerical schemes of finite element method for solving the continuum mechanics problems. Previously a method of acceleration of calculations was developed which uses the simplicial mesh inscribed in the original cubic cell partition of a three-dimensional body. In this paper we show that the obstacle to the construction of this design may be described in terms of homology groups modulo 2. The main goal of the paper is to develop a method of removing this obstacle. The reaching of the goal is based on efficient algorithms for computing bases of the homology groups which are dual with respect to the intersection form.

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INTRODUCTION

In recent decades, computing topology actively develops and becomes increasingly important in applications. In particular, a number of algorithms have been developed to efficiently compute homology groups, cycle intersection indices, search for minimal paths and cycles in given homology classes. The latter problems can be solved also with the help of bundles with discrete layers [1].

Applications include, for example, methods for removing topological defects of computer models. Topological methods are applied in the sensory networks modeling. To this end, we first construct an abstract simplicial complex by the Cech method using the fields of signal coverage from different sources, and then consider its homology groups [5, 6]. In paper [7] they discuss the topological characteristics of random fields that appear in numerical modeling of oil and gas collectors.

In [8–10] the authors proposed a new numerical scheme of the finite element method for solving three-dimensional problems of continuum mechanics. The scheme is proved to be effective. But the authors could guarantee its realizability only in the case when the investigated three-dimensional body is homeomorphic to the ball [11]. The purpose of this paper is to expand the scope of the scheme applicability. To reach this, we apply methods of algebraic and computational topology.

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