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On Complete Sublattices of Formations of Finite Groups

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Abstract—We prove that the lattice of all τ-closed saturated formations of finite groups is a complete sublattice of the lattice of all τ-closed solubly saturated formations of finite groups.

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INTRODUCTION

All groups in question are finite. It is known that the set of all formations $\mathcal{F}$ is a complete lattice with respect to the inclusion $\subseteq$. Recall that a nonempty set of formations Θ is called a *complete lattice of formations* ([1], P. 151) if the intersection of any family of formations from Θ belongs to Θ and there is a formation $\mathfrak{F}$ in Θ such that $\mathfrak{H} \subseteq \mathfrak{F}$ for any formation $\mathfrak{H}$ from Θ. Various families of formations can form complete lattices, in particular, the family of all saturated formations $\mathcal{L}$ and the family of all solubly saturated formations $\mathcal{C}$ ([1], P. 151; [2], P. 97).

It is well-known that a sublattice of a complete lattice $\mathcal{P}$ may be a complete lattice and be not a complete sublattice of $\mathcal{P}$ ([3], Chap. V, P. 195). A *sublattice* $\mathcal{H}$ of a complete lattice $\mathcal{P}$ is called *complete* if $\sup_{\mathcal{P}} \mathcal{X} \in \mathcal{H}$ and $\inf_{\mathcal{P}} \mathcal{X} \in \mathcal{H}$ for any nonempty subset $\mathcal{X} \subseteq \mathcal{H}$.

This being the case, we have $\sup_{\mathcal{H}} \mathcal{X} = \sup_{\mathcal{P}} \mathcal{X}$ and $\inf_{\mathcal{H}} \mathcal{X} = \inf_{\mathcal{P}} \mathcal{X}$.

The property of completeness for sublattices of formations was studied in [1], [4–6], [7] (P. 273). Note that the fact that sublattices of saturated and solubly saturated formations are complete was established due to functor methods in the study of formations (see A. N. Skiba's monograph [1]). A formation $\mathfrak{F}$ is called *saturated* if the condition $G/\Phi(G) \in \mathfrak{F}$ implies $G \in \mathfrak{F}$. A formation $\mathfrak{F}$ is called *solubly saturated* if the condition $G/\Phi(R(G)) \in \mathfrak{F}$ always implies $G \in \mathfrak{F}$. The symbol $R(G)$ denotes the greatest soluble normal subgroup of a group G. For a nonempty saturated formation $\mathfrak{F}$, it is accepted to write $\mathfrak{F} = LF(f)$ and say that $\mathfrak{F}$ is a saturated formation with local satellite f ([2], P. 20; [8], P. 356).

In [9], A. N. Skiba introduced multiply saturated and totally saturated formations. Every formation is considered to be 0-*tuply saturated*. For $n \geq 1$, a formation $\mathfrak{F}$ is called an n-*tuply saturated* if $\mathfrak{F} = LF(f)$, where all nonempty values of the local satellite f are $(n-1)$-tuply saturated formations. A formation is called *totally saturated* if it is n-tuply saturated for all natural numbers n.

Let $\tau(G)$ be a system of subgroups in a group G. It is said that τ is a *subgroup functor* (in the sense of A. N. Skiba, [1], P. 16) if the following conditions hold:

1) $G \in \tau(G)$;

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