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Approximate Solution to Integral Equation With Logarithmic Kernel of Special Form

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Abstract—Based on the quadrature formula with non-negative coefficients for integral with a special logarithmic kernel, we construct and substantiate a computational pattern for solving integral equation derived from the boundary-value problem for a function, which is harmonic in the unit disk under the boundary condition of the third kind. We obtain uniform estimates of deviations of the quadrature formula and the approximate solution to integral equation.

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Introduction. A boundary-value problem for a function $u = u(r, \varphi)$ harmonic in the unit disk under the boundary condition

$$\left(\frac{\partial u}{\partial r} + q(\varphi)u\right)\Big|_{r=1} = f(\varphi), \quad \varphi \in [-\pi, \pi], \tag{1}$$

where $q(\varphi)$ and $f(\varphi)$ are functions given on the segment $[-\pi, \pi]$, yields the integral equation ([1], P. 613) for boundary values of the desired function

$$u(\varphi) = \frac{1}{\pi}\int_{-\pi}^{\pi} q(\tau)u(\tau)\ln\left|t - e^{i\varphi}\right| d\tau - \frac{1}{\pi}\int_{-\pi}^{\pi} f(\tau)\ln\left|t - e^{i\varphi}\right| d\tau + C, \tag{2}$$

where $t = e^{i\tau}$, and C is a constant equal to the value of function u at the disk center.

The solution to Eq. (2) under the condition

$$\int_{-\pi}^{\pi} q(\varphi)u(\varphi)d\varphi = \int_{-\pi}^{\pi} f(\varphi)d\varphi \tag{3}$$

is equivalent to the solution to boundary-value problem.

One of the most efficient methods for approximate solution to integral equations is the reducing of integral equation to a system of linear algebraic equations. For the practical realization of such a method, the proof of solvability of this system and its stability is important.

1. Investigation of integral equation. Let us transform Eq. (2). Note that $\left|t - e^{i\varphi}\right| = 2\left|\sin\frac{\tau-\varphi}{2}\right|$, and due to mean theorem

$$C = u(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} u(\varphi)d\varphi,$$

from (2) under condition (3) we obtain the integral equation

$$u(\varphi) = \frac{1}{\pi}\int_{-\pi}^{\pi} q(\tau)u(\tau)\ln\left|\sin\frac{\tau-\varphi}{2}\right| d\tau - \frac{1}{\pi}\int_{-\pi}^{\pi} f(\tau)\ln\left|\sin\frac{\tau-\varphi}{2}\right| d\tau + \frac{1}{2\pi}\int_{-\pi}^{\pi} u(\tau)d\tau. \tag{4}$$

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