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On L_p-Convergence of Cesàro Means for Fourier Series with Monotonic Coefficients

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Abstract—For sine and cosine Fourier series with monotonic coefficients we study L_p-convergence $(1 < p < \infty)$ of their Cesàro means of a negative order.

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Introduction. Let $F(x) \in L(-\pi, \pi)$ and

$$\frac{a_0(F)}{2} + \sum_{k=1}^{\infty} a_k(F) \cos kx + b_k(F) \sin kx \tag{1}$$

be its Fourier–Lebesgue series. Denote by $\sigma_n^{\alpha}(x, F)$ the Cesàro (C, α)-means of (1). Convergence of Cesàro means for trigonometric Fourier series was investigated in [1–7]. By classical M. Riesz's result ([8], P. 94), for a continuous 2π-periodic function $F(x)$ and for every $\alpha > 0$, $\sigma_n^{\alpha}(x, F)$ converge to $F(x)$ uniformly on the real axis. Taberski [1] refined M. Riesz's theorem and established estimates for $\|\sigma_n^{\alpha}(x, F) - F(x)\|_C$ via the modulus of continuity of $F(x) \in C$. Approximation of $F(x) \in L_p(-\pi, \pi)$, $p \geq 1$ by Cesàro means of a positive order was investigated in details by Ul'yanov [2]. In particular, he found sharp estimates for $\|\sigma_n^{\alpha}(x, F) - F(x)\|_{L_p}$ via the modulus of continuity of $F(x) \in L_p(-\pi, \pi)$.

All these facts, as well as some others, show that Cesàro means of a positive order for trigonometric Fourier series have good approximative properties in the uniform metric and in the L_p-metric. For negative orders it does not hold; moreover, Men'shov [3] proved that there exists a function from L_2 such that every subsequence of Cesàro (C, α)-means, $\alpha \in (-1, -1/2)$, diverges on a set of positive measure. It is clear that the convergence of (C, α)-means, $\alpha \in (-1, -1/2)$, in L_p, $p \in (0, 2]$, for such functions is not the case. We should note that the function from the example by Men'shov has a sine series with positive coefficients.

In [4] they established a necessary and sufficient condition for summability in L_p of the Fourier series of $F \in L_p(-\pi, \pi)$, $p \in [1, 2]$, with the help of the (C, α)-method, $\alpha \in (-1, -1/2)$. The condition is $\sqrt{a_n(F)^2 + b_n(F)^2} = o(n^{\alpha})$, $n \to \infty$. We also point out [5] where, for negative α, some sufficient conditions of summability in $L_p(-\pi, \pi)$, $p \geq 1$, in the sense of the (C, α)-methods, of Fourier series and conjugate ones were found.

In connection with Men'shov' example, a natural question arises: For what values of α the Cesàro $(C, -\alpha)$ means of Fourier series with monotonic coefficients converge in L_p, $p > 1$? In other words, whether the monotonicity of Fourier coefficients improve approximative properties of Cesàro means in L_p, $p > 1$?

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