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On Generalization of Haar System and Other Function Systems in Spaces E_φ

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Abstract—We consider subsystems of system of Haar type and system of functions more general than the systems of contractions and displacements of one function. We obtain conditions under which these function systems are representation systems in spaces E_φ with certain restrictions on φ.

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1. About generalizations of the Haar system in spaces E_φ. Let Φ be a family of even, finite, non-decreasing on the half-line $[0,\infty)$ functions φ such that $\lim\limits_{t\to\infty}\varphi(t)=\varphi(\infty)=\infty$. Hereinafter we assume that for the function φ the conditions hold

$$\varphi(t)\in\Phi,\quad \varphi(0)=0,\quad \varphi(t)>0,\quad t>0,\quad \varphi(t)\in C[0,\infty). \qquad (*)$$

We denote by $\varphi(L)$ the set of all measurable functions $f(t)$ on $T=[0,1]$, for which $\int_T\varphi(f(t))dt<\infty$.

The set $\varphi(L)$ is also called a generalized Orlicz class ([1] pp. 1–5, 33). In the general case, the class $\varphi(L)$ is not linear. If we supplement the class $\varphi(L)$ by linearity, then we obtain the set $\varphi^*(L)$, in which one can introduce the quasi-norm (φ-norm) of elements with the help of the functional

$$\|f\|_\varphi=\inf\left\{u>0:\int_T\varphi\left(\frac{f(t)}{u}\right)dt<u\right\},\quad f\in\varphi^*(L),$$

so that $\varphi^*(L)$ becomes an F-space (the definition of F-space can be found in [1], P. 2). The set $\varphi^*(L)$ with the above introduced φ-norm is said to be the generalized Orlicz space ([1], pp. 1–5, 33). In this case, from the convergence with respect to the φ-norm the convergence with respect to the φ-distance follows for elements from the class $\varphi(L)$ ([1], pp. 1–5, 33).

We denote by E_φ the closure in $\varphi^*(L)$ of the set of bounded stepwise functions. The space E_φ is a separable F-space ([1], P. 36).

Representation systems (r. s.) were considered in papers [2–5], where one presented the definitions of r. s. in different spaces. As distinct from papers [6, 7], we do not investigate the best approximations and multi-dimensional decomposition by Haar type systems. The completeness by the measure of subsystems of the Haar system was considered in paper [8], where the following theorem has been obtained.

Theorem D ([8]). *Let $X=\{X_{n_k}\}$ be a subsystem of the Haar system. Let E_{n_k} be a carrier of X_{n_k} for any n_k and $E=\lim\limits_{k\to\infty}\sup\limits_{n_k}E_{n_k}=\bigcap\limits_{e=1}^{\infty}\bigcup\limits_{k=e}^{\infty}E_{n_k}$. Then X is a complete system by measure on the measurable set $G\subset[0,1]$ if and only if $\mu(G)=\mu(G\bigcap E)$.*

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