Optics

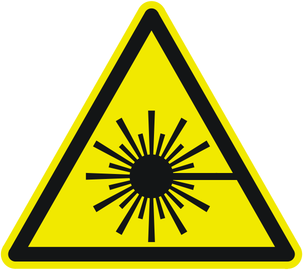
Photonic experiments

Optical Applications

LD
Physics
Leaflets

P5.8.3.4

Diffraction Gratings



Objects of the experiment

- Investigation the principle of gratings
- Measuring wavelength and grating constants
- Investigation spectral lamp and line spectrum

Safety notes

This experiment uses Lasers. According to EN 60825 they are rated class 3B.

474 5428:

Laser Class 3B, green 532nm, CW, <7 mW

Safety precautions are necessary. Please check with local regulations. Typically the use requires a safety sign and maybe a warning lamp that is on when the laser is activated and it might also be necessary to do and document a risk assessment. In some places it might be necessary to apply for a license or notify the relevant authorities.

Germany: According to OStrV it is necessary to inform the “Gewerbeaufsichtsamt” and the workers’ insurance “Berufsgenossenschaft” 14 days prior to startup.

Misuse of the lasers poses a health risk, especially for the eyes.

Do not operate the devices outside parameters specified in the manual.

People using the laser must be properly trained and students must be supervised.

As a general guidance, the user is advised to:

- Check the laser for damages before use
- Never to look into the beam
- Take necessary measures that no people or animals can accidentally enter the beam area
- Do not direct the beam on reflecting surfaces or into public areas
- Do not work close to the light path with reflecting tools
- Take off all jewelry and wristwatches when working with the laser to avoid reflections
- While placing or removing optical parts in the light path, switch off the laser or cover its exit
- Use laser protection glasses or laser adjustment glasses where necessary
- Supervise students by trained personnel when they work with the laser system
- Use the laser system only as described in the instruction manuals

Principles

a) Principle of diffraction

Diffraction is the bending of waves as they pass by some objects or through an aperture. The phenomenon of diffraction can be understood using Huygens’s principle which states that:

Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.

According to Huygens’s principle, light waves incident on two slits will spread out and exhibit an interference pattern in the region beyond. The pattern is called a diffraction pattern.

PJD 2014-12

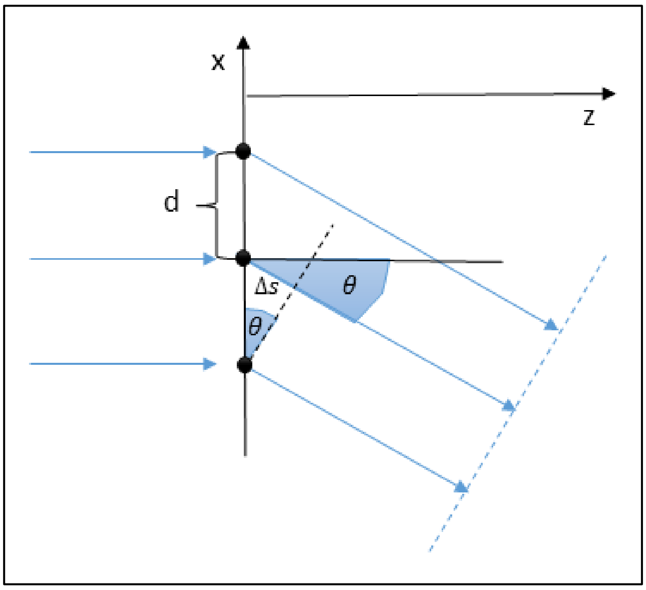


Fig. 1: Principle of diffraction. Calculating the optical path difference Δs.

A diffraction grating is a collection of reflecting (or transmitting) elements separated by a distance comparable to the wavelength of light under study, as shown in Fig. 1. It may be thought of as a collection of diffracting elements, such as a pattern of transparent slits in an opaque screen, or a collection of reflecting grooves on a substrate. Upon diffraction, an electromagnetic wave incident on a grating will have its electric field amplitude, or phase, or both, modified in a predictable manner, due to the periodic variation in refractive index in the region near the surface of the grating.

To calculate the diffraction of a parallel wave through a slit, assume that there are *N* regularly ordered sets of oscillators with the distance *d* to each other. If a parallel wave hits the slit, all oscillators begin to oscillate at the same time.

Consider that under a special angle θ the waves have to travel different distances. The optical path difference Δs is illustrated in Fig. 1 and can be calculated with the following formula:

$$\Delta s = d \cdot \sin\theta$$

This causes a phase difference of

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta s = \frac{2\pi}{\lambda} d \cdot \sin\theta$$

of two neighbored waves. When this distance is equal to an integer number m of wavelengths λ of the incident light, the two beams are in phase and will exhibit constructive interference by displaying a series of bright regions on the screen. These interference maxima are given by:

$$d\sin\theta = m\lambda, \quad m = 0, \pm1, \pm2, \ldots$$

where λ is the wavelength, d is the grating spacing, and m is an integer called the order number. If the path difference between adjacent beams is $\left(m + \frac{1}{2}\right)\lambda$, then destructive interference will result in dark regions, or interference minima, on the screen. The zero-order beam $m = 0$ is a continuation of the incident beam. Hence the measurement of the angle, together with the order number m, gives the ratio λ/d, and if either λ or d is known, the other can be calculated.

The intensity of the diffracted light of a diffraction grating is given by the following formula:

$$I(\theta) = I_o \frac{\sin^2\left[\pi\left(\frac{b}{\lambda}\right)\sin\theta\right]}{\left[\pi\left(\frac{b}{\lambda}\right)\sin\theta\right]^2} \cdot \frac{\sin^2\left[N\pi\left(\frac{d}{\lambda}\right)\sin\theta\right]}{\sin^2\left[\pi\left(\frac{d}{\lambda}\right)\sin\theta\right]}$$

b is the width of a single slit, d is the distance between two slits. N is the number of slits. θ is the angular distance from the center of the diffraction pattern and λ is the wavelength of the light.

The first factor represents the diffraction of a single slit and the second factor the interference between N slits.

b) Spectral resolution

The resolving power R of a diffraction grating is at best an impractical, theoretical concept and is given by:

$$R = \frac{\lambda}{\Delta\lambda}$$

where $\Delta\lambda$ is the difference in wavelength between two spectral lines of equal intensity. Resolution is the ultimate ability of an instrument to separate two spectral lines. By the Raleigh criterion, two peaks are considered resolved when the maximum of one falls on the first minimum of the other. It can be shown that:

$$R = \frac{\lambda}{\Delta\lambda} = knW_g = kN$$

where λ, the central wavelength to be resolved; W_g, the illuminated width of the grating; and N, the total number of grooves on the illuminated width of the grating.

c) Spectral lamp

In general there are three classes of lamps: incandescent, discharge, and solid-state lamps. Incandescent lamps produce light by heating a filament until it glows. Discharge lamps produce light by ionizing a gas through electric discharge inside the lamp. Solid-state lamps use either a phenomenon called electroluminescence to convert electrical energy directly to light or pn-junction diodes that can emit light at a special wavelength when a special voltage is applied.

Spectral lamps are discharge lamps that produce light by passing an electric current through a gas that emits light when ionized by the current. An auxiliary device known as a ballast supplies voltage to the lamp's electrodes, which have been coated with a mixture of alkaline earth oxides to enhance electron emission. Two general categories of discharge lamps are used to provide illumination: high-intensity discharge and fluorescent lamps.

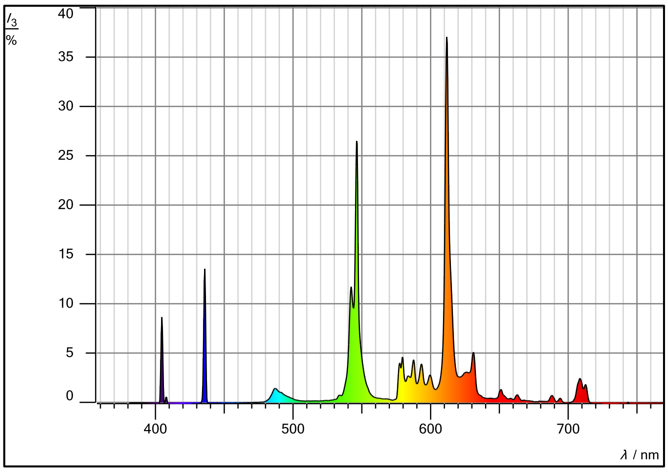


Fig. 2: Spectrum of mercury lamp.

The fluorescent lamp is a gas discharge source that contains mercury vapor at low pressure, with a small amount of inert gas for starting. Once an arc is established, the mercury vapor emits ultraviolet radiation. Fluorescent powders (phosphors) coating the inner walls of the glass bulb respond to this ultraviolet radiation by emitting wavelengths in the visible region of the spectrum. In this experiment a fluorescent lamp is used and should be investigated.

Experiments

To visualize and measure the properties of transmission gratings, spectral lamps and line spectrums the following experiments are presented.

Apparatus

1	Plano-Convex lens f = 40 mm, C25 mount	474 5216
1	Biconvex lens f = 60 mm, C25 Mount	474 5256
1	Beam expander 6x	474 5263
1	Beam expander 2.7x	474 5264
1	Transmission gratings, Set of 5	474 5268
1	Optical Screen with XY Scale	474 6417
1	Photodetector signal conditioning box	474 306
1	Si PIN Photodetector	474 321
1	Digital multimeter DMM 121	531 173
1	Screened cable, BNC/4 mm	575 24
1	Spectral Lamp with Slit and Power Supply	474 5417
1	Diode Laser Module, 532 nm	474 5418
1	Profile rail, 500 mm	474 5442
1	Swivel Unit with Carrier	474 121
2	Mounting plate 40, C25	474 6411
1	Mounting Plate C25 with Carrier 20 mm	474 209
1	Adjustment holder, 4 axes, with stop ang	474 2112
1	Transport and Storage Box #01	474 251
1	Manual Diffraction gratings	474 7213

Experiment 1:

In this experiment the wavelength of a laser is investigated by using different transmission gratings.

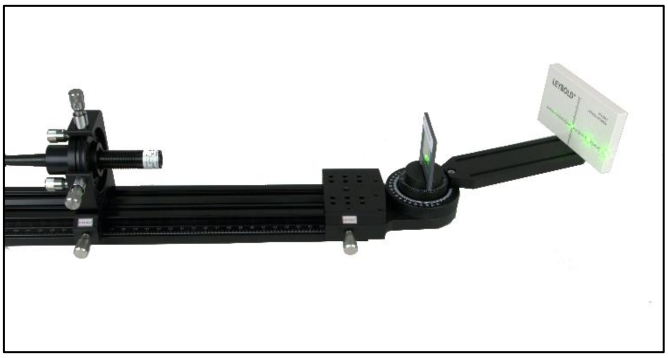


Fig. 3: Setup to measure the diffraction angle

Setup on Bench:

1	Profile rail, 500 mm	474 5442
1	Swivel Unit with Carrier	474 121
1	Adjustment holder, 4 axes, with stop ang	474 2112
1	Diode Laser Module, 532 nm	474 5418
1	Transmission gratings, Set of 5	474 5268
1	Optical Screen with XY Scale	474 6417

On the bench, the diode laser module is put into the adjustment holder. Next, set the swivel unit with carrier on the bench in straight direction ($\theta = 0°$) and place the optical screen with XY scale at the end of it. Insert the different transmission gratings to the middle of the swivel unit. Turn the grating so that the reflex is at the output window of the diode laser.

Measure the angles to the different diffraction orders by turning the swivel unit. By measuring the positive and negative diffraction orders it is possible to calibrate the zero order to zero degrees.

Results:

The measured positive and negative angle has been averaged over. With the given numbers of slits per mm per grating, we can calculate the grating constant:

$$d = \frac{0.001}{80}\,\mathrm{m} = 1.25 \times 10^{-5}\,\mathrm{m}$$

The wavelength is then given by:

$$\lambda = \frac{d \sin\theta}{m}$$

This leads to the following tables:

Table 1: Measured diffraction with a 80 lines / mm grating

Order	θ in °	λ in nm	$\Delta\lambda$ in nm
0	0		
1	2	436	217
2	5	545	108
3	7.5	544	72
4	10	543	54
5	12	520	43
6	15	539	35

7	18	552	30
8	20	534	25
9	23	543	22
10	25	528	20
11	28	533	18
12	30	521	16
13	33	524	14
14	36	525	13
15	40	536	11
16	43.5	538	10
17	47	538	9
18	50	532	8
19	54.5	536	7

The average wavelength is $\lambda = 530\ \mathrm{nm}$, which is very close to the assumed wavelength $\lambda = 532\ \mathrm{nm}$.

At this point it is possible to calculate an error with the propagation of uncertainty. Therefore calculate the partial derivative of λ with respect to θ.

$$\Delta\lambda = \frac{d\cos\theta}{m} \cdot \Delta\theta$$

With this setup, it is possible to measure only in one degree steps. Therefore assume an error of $\Delta\theta = 1°$. This can be done for every single transmission grating. These leads to the following average wavelengths shown in table 2.

Table 2: Average wavelength to the different gratings with standard deviation.

Grating in lines/mm	$\boldsymbol{\lambda_A}$ **in nm**	**Δλ in nm**
80	530	38
100	544	30
300	538	20
600	546	14
1200	524	11

This leads to a mean wavelength of

$$\lambda = (536 \pm 23)\mathrm{nm},$$

which is with the error bars within the given value of 532 nm.

Experiment 2:

In this experiment the intensities of the grating maxima are investigated by using different transmission gratings.

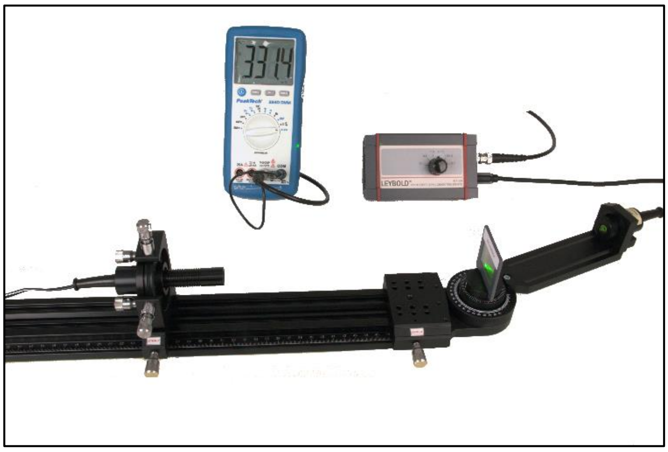


Fig. 4: Setup to measure the intensities of the grating maxima.

Setup on Bench:

1	Profile rail, 500 mm	.... 474 5442
1	Swivel Unit with Carrier	.... 474 121
1	Adjustment holder, 4 axes, with stop ang	..... 474 2112
1	Diode Laser Module, 532 nm	.... 474 5418
1	Transmission gratings, Set of 5	.... 474 5268
1	Photodetector signal conditioning box	...... 474 306
1	Si PIN Photodetector	.... 474 321
1	Digital multimeter DMM 121	.... 531 173
1	Screened cable, BNC/4 mm	.... 575 24

Replace the optical screen with Si PIN Photodetector on the mounting plate C25. At the end of the bench the photodetector is connected with the photodetector signal conditioning box and with the digital multimeter via BNC cable. Run the multimeter in voltage mode and set the controller of the conditioning box so that the output voltage is around about a few hundred mV. Note that the conditioning box runs with a 9 V battery so that this is the maximum output voltage. Measure the intensities of the maxima at the different orders.

Results:

In Fig. 5 is an example of a measured intensity distribution of a transmission grating with 300 lines per mm.

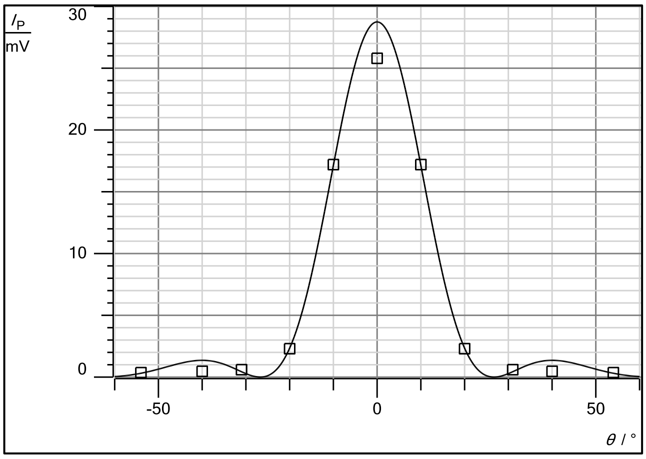


Fig. 5: Intensity of 300 lines / mm transmission grating with a Frauenhofer fit.

While the multi-slit grating determines the sharp peaks of each diffraction order, the envelope of the individual maxima amplitudes is given by the fraunhofer diffraction pattern of one individual slit.

$$I(\theta) = I_0[\mathrm{sinc}(\beta)]^2$$

where

$$\beta = \frac{\pi b}{\underbrace{\lambda}_{B}} \sin\theta$$

Therefore we can measure independently a whole series of maximum amplitudes and fit a fraunhofer pattern to it, yielding the (average) width of each slit, as shown in Fig. 5. The grating constant, i.e. the distance between two slits can be calculated from the n-th maximum angular position and the wavelength used.

The fit leads to

$$B = 1.98$$

With a given wavelength $\lambda = 532$ nm we get following average width of each slit

$$b = 3.35 \times 10^{-7}\ \mathrm{m}.$$

Using both the width of the slit and the separation of the slits, we know that the grating was made with 40.2 % transmissive bars and 59.8 % black bars.

Experiment 3:

In this experiment a spectral lamb is investigated by using the 1200 lines/mm transmission grating.

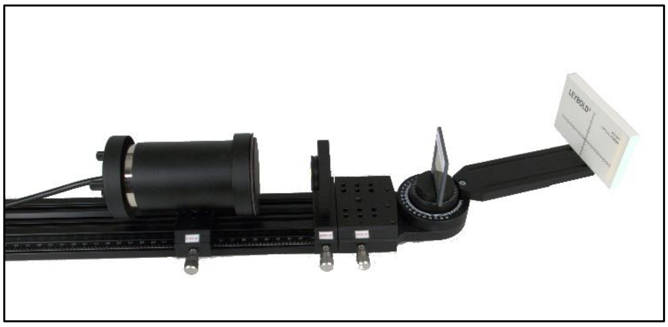


Fig. 6: Setup for investigating the spectral lamb.

Setup on Bench:

1	Profile rail, 500 mm	474 5442
1	Swivel Unit with Carrier	474 121
1	Spectral Lamp with Slit and Power Supply	474 5417
1	Mounting Plate C25 with Carrier 20 mm	474 209
1	Biconvex lens f = 60 mm, C25 Mount	474 5256
1	Transmission gratings, Set of 5	474 5268
1	Optical Screen with XY Scale	474 6417

Replace the adjustment holder and laser module with the spectral lamb. There are two different slits to place directly in front of the spectral lamb. Note the spectral lamb can be moved in the case so that the slit fits perfect at the end. Use the thinner slit for this experiment. Put the transmission grating with 1200 lines per mm in the holder and place the optical screen at the end of the swivel unit. Place the lens on the mounting plate with carrier so that at the optical screen so that there is a sharp image of the slit. Measure the angles to the different lines. Therefor turn the swivel unit to positive and negative angles. Place the y-axis as in middle of the lines as possible.

Results:

The measured angles are listed in the following table.

Line	**Angle θ in $^\circ$**
Purple	29
Blue	31
Turquoise	35
Green	40
Yellow	43
Red	45.5

With the given numbers of slits per mm (1200 lines/mm), we can calculate the grating constant:

$$d = \frac{0.001}{1200}\ \mathrm{m} = 8.33 \times 10^{-7}\ \mathrm{m}$$

The colored lines are maxima of the first order so that the wavelength can be calculated by the following formula:

$$\lambda = d \sin\theta$$

This leads to the following table

Line	**λ in nm**
Purple	404
Blue	429
Turquoise	477
Green	536
Yellow	568
Red	594