Semiclassical ultraextremal black holes

Arkady Popov
in collaboration with O. Zaslavskii

Institute of Mathematics and Mechanics, Kazan Federal University
Kazan, Russia

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We consider quantum backreaction of the quantized scalar field with an arbitrary mass and curvature coupling on ultraextremal horizons. The problem is distinguished in that (in contrast to non-extremal or extremal black holes) the WKB approximation remains valid near r_+ (which is the radius of the horizon) even in the massless limit. We examine the behavior of the stress-energy tensor of the quantized scalar field near r_+ and show that quantum-corrected objects under discussion do exist.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (1)$$

where

$$f(r_+) = 0 \quad (2)$$

$$f(r) = \frac{1}{n!}\frac{d^n f}{dr^n}_{|r=r_+}(r - r_+)^n + O\left((r - r_+)^{n+1}\right) \quad (3)$$

$n = 2$ – extremal horizon

$n > 2$ – **ultraextremal horizon**

Hawking radiation temperature

$$T = \frac{f'}{4\pi f}\Bigg|_{r=r_+} \qquad (G = c = \hbar = 1) \quad (4)$$

The considered case (classical background)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (5)$$

$$f(r) = \frac{(r + 3r_+)}{6r^2 r_+^2} (r_+ - r)^3 \qquad (6)$$

$$\Lambda {r_+}^2 = \frac{Q^2}{{r_+}^2} = \frac{1}{2}$$

Romans, *Nucl. Phys.* **B383** (1992)

Q - charge of black hole

Λ - cosmological constant

The classical background is

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{7}$$

where

$$f(r) = \frac{(r+3r_+)}{6r^2 \mathring{r}_+{}^2}\left(\mathring{r}_+ - r\right)^3. \tag{8}$$

The quantum corrected metric reads

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{V(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{9}$$

$$V(\mathring{r}_+ + \Delta r) = 0, \tag{10}$$

$$U(\mathring{r}_+ + \delta r) = 0. \tag{11}$$

The role of a classical source in Einstein's equations is played by an electromagnetic field

$$T_{\nu}^{\mu\,(cl)} = \frac{Q^2}{r^4} diag(-1,-1,1,1). \qquad (12)$$

In that case the nontrivial Einstein's equations

$$G_{\nu}^{\mu} + \Lambda \delta_{\nu}^{\mu} = 8\pi \left(T_{\nu}^{\mu\,(cl)} + \langle T_{\nu}^{\mu} \rangle_{ren} \right) \qquad (13)$$

are

$$\frac{V'}{r} + \frac{V}{r^2} - \frac{1}{r^2} + \Lambda = 8\pi \left(\langle T_t^t \rangle_{ren} - \frac{Q^2}{r^4} \right), \qquad (14)$$

$$\frac{VU'}{rU} + \frac{V}{r^2} - \frac{1}{r^2} + \Lambda = 8\pi \left(\langle T_r^r \rangle_{ren} - \frac{Q^2}{r^4} \right), \qquad (15)$$

$$\frac{VU''}{2U} - \frac{VU'^2}{4U^2} - \frac{V'U'}{4U} \frac{VU'}{2rU} + \frac{V'}{2r} + \Lambda = 8\pi \left(\langle T_\theta^\theta \rangle_{ren} + \frac{Q^2}{r^4} \right). \qquad (16)$$

To evaluate the role of backreaction, we start not from the classical background with further adding quantum corrections but from the quantum-corrected self-consistent geometries from the very beginning

$$\begin{aligned} U(r_+) &= U'(r_+) = U''(r_+) = 0, \\ V(r_+) &= V'(r_+) = V''(r_+) = 0. \end{aligned} \tag{17}$$

The solutions of equations (14,15) can be written as follows

$$V(r) = 1 + \left(\frac{\Lambda r_+^3}{3} - \frac{Q^2}{r_+} - r_+ \right) \frac{1}{r} - \frac{\Lambda r^2}{3} + \frac{Q^2}{r^2} + \frac{8\pi}{r} \int_{r_+}^{r} d\tilde{r} \tilde{r}^2 \langle T_t^t \rangle_{ren}, \tag{18}$$

$$U(r) = V(r) \left\{ 1 + const + 8\pi \int_{r_+}^{r} d\tilde{r} \tilde{r} \frac{\langle T_r^r \rangle_{ren} - \langle T_t^t \rangle_{ren}}{V(\tilde{r})} \right\}, \tag{19}$$

where $\langle T_\nu^\mu \rangle_{ren}$ is calculated on the unperturbed background.

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{V(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{20}$$

$$Q^2 = \frac{r_+^2}{2} - 2\pi r_+^5 \frac{d\langle T_t^t\rangle_{ren}}{dr}_{|r=r_+} \tag{21}$$

$$\Lambda = \frac{1}{2r_+^2} + 8\pi\langle T_t^t\rangle_{ren\,|r=r_+} + 2\pi r_+ \frac{d\langle T_t^t\rangle_{ren}}{dr}_{|r=r_+} \tag{22}$$

$$V(r) = \left[-\frac{2}{3r_+^3} + \frac{20\pi}{3}\frac{d\langle T_t^t\rangle_{ren}}{dr}_{|r=r_+} + \frac{4\pi r_+}{3}\frac{d^2\langle T_t^t\rangle_{ren}}{dr^2}_{|r=r_+}\right](r-r_+)^3 + O\left((r-r_+)^4\right), \tag{23}$$

$$U(r) = \left[-\frac{2(1+const)}{3r_+^3} + \frac{20\pi}{3}\frac{d\langle T_t^t\rangle_{ren}}{dr}_{|r=r_+} + \frac{4r_+\pi}{3}\frac{d^2\langle T_t^t\rangle_{ren}}{dr^2}_{|r=r_+}\right](r-r_+)^3 + O\left((r-r_+)^4\right) \tag{24}$$

The well-known DeWitt-Schwinger approximation of $\langle T^{\mu}_{\nu}\rangle_{ren}$ is expansion with respect to the small parameter

$$\epsilon = \frac{1}{m^2 L^2} \ll 1 \tag{25}$$

m - mass of the quantized field,
L - characteristic curvature scale of the gravitational background.

Anderson, Hiscock, Samuel, *Phys. Rev. D* **51** (1995);
Matyjasek, *Phys. Rev. D* **61** (2000), *Phys. Rev. D* **63** (2001);
Popov, *Phys. Rev. D* **64** (2001)

The small parameter of WKB expansion near ultraextremal horizon for the arbitrary mass of scalar field

$$\varepsilon_{\mathit{WKB}} = \frac{(r - r_+)}{r_+^3\left(m^2 + \dfrac{2\xi}{r_+^2}\right)} \ll 1 \tag{26}$$

It has been calculated the next terms of the expectation value of the stress-energy tensor operator of the quantized scalar field

$$\langle T^\mu_\nu\rangle_{ren}=\langle T^\mu_\nu\rangle_{ren}{}_{|r=r_+} + \frac{d\langle T^\mu_\nu\rangle_{ren}}{dr}_{|r=r_+}(r-r_+) + \frac{d^2\langle T^\mu_\nu\rangle_{ren}}{2!\ dr^2}_{|r=r_+}(r-r_+)^2 + \frac{d^3\langle T^\mu_\nu\rangle_{ren}}{3!\ dr^3}_{|r=r_+}(r-r_+)^3 + O\left((r-r_+)^4\right) \qquad (27)$$

The zeroth-order terms are Popov, *Phys. Rev. D* **64** (2001)

$$\langle T_t^t\rangle_{ren|r=r_+}=\langle T_r^r\rangle_{ren|r=r_+} = \frac{1}{\pi^2 r_+^4}\left\{\left(\xi-\frac{1}{8}\right)\frac{m^2r_+^2}{32}+\frac{3\xi^2}{32}-\frac{11\xi}{384}+\frac{79}{30720}\right.$$
$$+\left[-\frac{m^4r_+^4}{64}-\left(\xi-\frac{1}{6}\right)\frac{m^2r_+^2}{16}-\frac{\xi^2}{16}+\frac{\xi}{48}-\frac{1}{480}\right]\ln\left(\frac{\mu_+^2}{m_{DS}^2r_+^2}\right)$$
$$\left.+\frac{\mu_+^4}{8}\left[I_1(\mu_+)-I_2(\mu_+)\right]\right\}, \tag{28}$$

$$\langle T_\theta^\theta\rangle_{ren}=\langle T_\varphi^\varphi\rangle_{ren} = \frac{1}{\pi^2 r_+^4}\left\{\frac{m^2r_+^2}{32}\left(\xi-\frac{1}{8}\right)-\frac{1}{32}\left(\xi-\frac{1}{8}\right)^2\right.$$
$$+\left[-\frac{m^4r_+^4}{64}+\frac{\xi^2}{16}-\frac{\xi}{48}+\frac{1}{480}\right]\ln\left(\frac{\mu_+^2}{m_{DS}^2r_+^2}\right)$$
$$\left.-\left(\xi-\frac{1}{8}\right)\frac{\mu_+^2}{4}I_1(\mu_+)+\frac{\mu_+^4}{8}I_2(\mu_+)\right\}, \tag{29}$$

where
$$\mu_+^2 = m^2r_+^2+2\xi-1/4>0, \tag{30}$$
$$I_n(\mu_+) = \int_0^\infty \frac{x^{2n-1}\ln|1-x^2|}{1+e^{2\pi|\mu_+|x}}dx, \tag{31}$$

$$m = 0, \quad \xi = 1/6 \qquad (G = c = \hbar = 1)$$

$$\begin{aligned}\langle T_t^t\rangle_{ren} \simeq & \frac{1}{\pi^2 r_+^4}\left[0.00078 + \frac{1}{2880}\ln\left(m_{\boldsymbol{DS}}^2 r_+^2\right)\right] + \frac{1}{\pi^2 r_+^5}\left[-0.00241\right. \\ & \left. - \frac{1}{720}\ln\left(m_{\boldsymbol{DS}}^2 r_+^2\right)\right](r - r_+) + \frac{1}{\pi^2 r_+^6}\left[0.00463 + \frac{1}{288}\ln\left(m_{\boldsymbol{DS}}^2 r_+^2\right)\right](r - r_+)^2 \\ & + \frac{1}{\pi^2 r_+^7}\left[0.00417 - \frac{1}{90}\ln\left(m_{\boldsymbol{DS}}^2 r_+^2\right)\right](r - r_+)^3 + O\left(\frac{(r - r_+)^4}{r_+^8}\right), \end{aligned} \tag{32}$$

$$\begin{aligned}\langle T_r^r\rangle_{ren} - \langle T_t^t\rangle_{ren} \simeq & \frac{1}{\pi^2 r_+^7}\left[-0.007406 + \frac{1}{360}\ln\left(m_{\boldsymbol{DS}}^2 r_+^2\right)\right](r - r_+)^3 \\ & + O\left(\frac{(r - r_+)^4}{r_+^8}\right), \end{aligned} \tag{33}$$

$$\langle T_\theta^\theta \rangle_{ren} \simeq \frac{1}{\pi^2 r_+^4}\left[-0.00043 - \frac{1}{2880}\ln\left(m_{DS}^2 r_+^2\right)\right]$$
$$+\frac{1}{\pi^2 r_+^5}\left[0.00102 + \frac{1}{720}\ln\left(m_{DS}^2 r_+^2\right)\right](r-r_+)$$
$$+\frac{1}{\pi^2 r_+^6}\left[-0.00115 - \frac{1}{288}\ln\left(m_{DS}^2 r_+^2\right)\right](r-r_+)^2$$
$$+O\left(\frac{(r-r_+)^3}{r_+^7}\right). \tag{34}$$

m_{DS} is equal to the mass m of the field for a massive scalar field. For a massless scalar field it is an arbitrary parameter due to the infrared cutoff in renormalization counterterms for $\langle T_\nu^\mu \rangle$. A particular choice of the value of m_{DS} corresponds to a finite renormalization of the coefficients of terms in the gravitational Lagrangian and must be fixed by experiment or observation.

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{V(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (35)$$

$$V(r) = \left[-\frac{2}{3r_+^3} - \frac{0.00372}{\pi r_+^5}\right](r-r_+)^3 + \left\{\frac{7}{6r_+^4} - \frac{20}{3\pi r_+^6}\left[0.03058 - \frac{1}{120}\ln\left(m_{DS}^2 r_+^2\right)\right]\right\}(r-r_+)^4 + O\left(\frac{(r-r_+)^5}{r_+^5}\right), \quad (36)$$

$$U(r) = \left[-\frac{2}{3r_+^3} - \frac{0.00372}{\pi r_+^5}\right](r-r_+)^3 + \left\{\frac{7}{6r_+^4} + \frac{1}{\pi r_+^6}\left[-0.26311 + \frac{7}{90}\ln\left(m_{DS}^2 r_+^2\right)\right]\right\}(r-r_+)^4 + O\left(\frac{(r-r_+)^5}{r_+^5}\right). \quad (37)$$

Hawking radiation temperature

$$T = \frac{U'}{4\pi}\sqrt{\frac{V}{U}}\Bigg|_{r=r_+} = 0 \qquad (G = c = \hbar = 1) \quad (38)$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{V(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$V(r)=\left[-\frac{2}{3r_+^3} + \frac{1}{m^2\pi r_+^7}\left(-\frac{\xi}{135} + \frac{1}{378}\right)\right](r-r_+)^3 + \left[\frac{7}{6r_+^4} + \frac{1}{m^2\pi r_+^8}\left(\frac{\xi}{60} - \frac{1}{360}\right)\right](r-r_+)^4 + O\left(\frac{(r-r_+)^5}{r_+^5}\right),$$

$$U(r)=\left[-\frac{2(1+const)}{3r_+^3} + \frac{1}{m^2\pi r_+^7}\left(-\frac{\xi}{135} + \frac{1}{378}\right)\right](r-r_+)^3 + \left[\frac{7(1+const)}{6r_+^4} + \frac{1}{m^2\pi r_+^8}\left(\frac{5\xi}{108} - \frac{149}{7560}\right)\right](r-r_+)^4 + O\left(\frac{(r-r_+)^5}{r_+^5}\right).$$

Hawking radiation temperature

$$T = \frac{U'}{4\pi}\sqrt{\frac{V}{U}}\Bigg|_{r=r_+} = 0 \qquad (G = c = \hbar = 1) \tag{39}$$

Conclusion

- WKB approximation for $\langle T^{\mu}_{\nu} \rangle_{ren}$ is valid near ultraextremal horizon for any finite mass m of the quantized field, including $m = 0$.
- Semiclassical (quantum-corrected) ultraextremal horizons exist for any mass of the field.