ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 9, pp. 1–8. © Allerton Press, Inc., 2015.
Original Russian Text © A.V. Vasin, 2015, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2015, No. 9, pp. 3–13.

On Bases which Contain Functions Dependent on Five Variables with Unreliability Coefficient 1

A. V. Vasin[1*]

[1] *Penza State University, ul. Krasnaya 40, Penza, 440026 Russia*

Received November 13, 2013

Abstract—We consider realization of Boolean functions by circuits composed of unreliable functional elements in some complete finite basis. We assume that all elements are subjected independently of each other to inverse failures at the output. We find a set of functions depending on five variables. We prove that unreliability coefficient of a basis which contains functions of referred set equals one.

DOI: 10.3103/S1066369X15090017

Keywords: *unreliable functional elements, circuits asymptotically optimal with respect to reliability, inverse failures on outputs of elements, synthesis of circuits composed of unreliable elements.*

We consider the Boolean functions realization by circuits (see, e.g., [1] or [2]) of unreliable function elements in the complete finite basis B. We say that the circuit S of unreliable elements realizes the Boolean function $f(x_1, x_2, \ldots, x_n)$ if we obtain $f(\mathbf{a})$ as an output of the circuit S on the binary circuit input $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ under no failure.

The first considerations of the reliable circuits synthesis from the unreliable function elements is due to J. von Neumann [3]. He assumed that all the circuit elements are mutually independently subject on output to some inverse failures with probability ε ($\varepsilon \in (0, 1/2)$). These failures work as follows: in the case of good condition the function element realizes the Boolean function φ attributed to it and in the other case this element produces the function $\overline{\varphi}$. J. von Neumann applied the iteration technique to prove that in an arbitrary complete finite basis any Boolean function can be realized by some circuit whose wrong output probability under any input does not exceed $c_1\varepsilon$ (here c_1 is some constant depending on the basis) for $\varepsilon \in (0, 1/6]$.

Two important parameters describe the unreliable element circuit: the circuit error output probability and the circuit complexity. The main drawback of the von Neumann method is that the iterations number increase also exponentially increases the circuit complexity (the rough estimate is 3^k, here k is the iterations number). So we used to consider the circuit complexity as the main investigation object [4–6].

Here we pay our attention more to the maximal output error probability for the circuit optimally or "almost" optimally realizing the function from the error probability viewpoint. We also consider the bases whose unreliability coefficient equals 1. Let us introduce the necessary notions.

The circuit S *unreliability* $P(S)$ is the maximal error probability on the circuit S output under all possible input combinations. Naturally the circuit S *reliability* equals $1 - P(S)$.

Put $P_\varepsilon(f) = \inf\limits_S P(S)$, here we consider the infimum over all the circuits S of unreliable elements realizing the Boolean function $f(x_1, x_2, \ldots, x_n)$. The circuit A of unreliable elements realizing the function f is said to be *asymptotically optimal* (the asymptotically best one) with respect to reliability if $P(A) \sim P_\varepsilon(f)$ for $\varepsilon \to 0$, i.e., $\lim\limits_{\varepsilon\to 0} \frac{P_\varepsilon(f)}{P(A)} = 1$.

The reliability asymptotically optimal circuits construction problem over the complete bases of two-input elements can be found in the work by M. A. Alekhina [7]. The solution was constructed under the

*E-mail: alvarvasin@mail.ru.