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Solving the Inverse Mixed Boundary-Value Problem of Aerohydrodynamics in a New Statement

R. B. Salimov[1*]

[1] *Kazan State Architecture and Civil Engineering University ul. Zelyonaya 1, Kazan, 420043 Russia*

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Abstract—We consider the following inverse mixed boundary-value problem of aerohydrodynamics. It is required to find a form of an airfoil circulated by a potential flow of an incompressible non-viscous liquid. A part of the profile is known; it is a broken line, convex upwards. The other part is found by the values of velocity potential, given as a function of parameter which can be either the abscissa, or the ordinate of an airfoil point. On a small part of the airfoil, containing the leading edge, the parameter is the ordinate, and for other points of the unknown part it is the abscissa.

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Let L_z be an airfoil in the plane of complex variable $z = z + iy$, circulated by a potential flow of an incompressible non-viscous liquid, with complex potential $w = w(z) = \varphi + i\psi$ and velocity of unperturbed flow $\nu_\infty e^{i\eta_\infty}$, $\nu_\infty > 0$, $\frac{\pi}{2} < \eta_\infty < \frac{3\pi}{2}$. Let $x = x_B = 0$ be the abscissa of the trailing edge B of L_z, and $x = x_D > 0$ be the abscissa of the point D of L_z, the most remote from the imaginary axis (the leading edge of L_z); for all other points of L_z we have $0 < x < x_D$.

Denote by D_z the domain exterior to L_z (the flow area). We will assume that everywhere in D_z the derivative $w'(z) = \nu e^{-i\eta}$ is finite and does not vanish. Here ν is the module of the vector of velocity at the point $z = x + iy$, and η is the angle of inclination of the vector.

We will assume that $\psi = 0$ on L_z, the upstream stagnation point A of the flow is on the lower part of L_z, and the velocity potential at the point $\varphi = \varphi_A = 0$. We will mean that the trailing edge B coincides with the downstream critical point. We also assume that the velocity potential φ on L_z is a continuous function on L_z excluding the point B. We will denote by $\varphi = \varphi_B$ and $\varphi = \varphi_H$, $\varphi_B > \varphi_H > 0$, the limiting values of the velocity potential φ at the point B as we go to it by the upper and lower parts of L_z; the difference $\varphi_B - \varphi_H = \Gamma$ is the velocity circulation along L_z.

Note that the function $w = w(z)$ maps conformally the domain D_z with a slit, going along a curve lying outside L_z and connecting B with $z = \infty$, onto the domain D_w in the plane of $w = \varphi + i\psi$, cut along the positive part of the real axis, beginning at the point A corresponding to $w = 0$. The arc AB on the lower surface of L_z matches the segment lying on the upper edge of the slit specified above, and on the segment we have $0 < \varphi < \varphi_H$. The arc ADB of L_z matches the segment lying on the lower edge of the indicated slit, and on the segment the inequality $0 < \varphi < \varphi_B$ holds.

Below C^+ and C^- are the points of the upper and lower surfaces of L_z with abscissa $x_C = x_{C^+} = x_{C^-}$, and K is a point lying on the arc BC^- of L_z with abscissa x_K.

Consider the following inverse mixed boundary-value problem.

Find an airfoil L_z, by known values of the velocity potential φ, if on arcs BC^+ and KC^- it is given as a function of the abscissa x of a point $z = x + iy$ of the curve L_z:

$$\begin{aligned} \varphi &= \varphi^+(x), \quad 0 \le x \le x_C, \\ \varphi &= \varphi^-(x), \quad x_K \le x \le x_C. \end{aligned} \tag{1}$$

[*] E-mail: salimov.rsb@gmail.com.