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On Summary Equation Generated by a Quadrangle

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Abstract—We investigate a four-element functional equation in a class of solutions holomorphic outside a quadrangle and vanishing at infinity. We propose equivalent regularization method of such quadrangles. We construct examples with only single condition of solvability for such an equation. We show applications to moments problem for entire functions of exponential type.

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INTRODUCTION

Let D be a quadrangle with vertices t_j, $j = \overline{1,4}$ and sides l_j which are listed in the order of circumvention of the positively oriented border $\Gamma = \partial D$ (l_1 is an interval with ends t_1 and t_2). Transformations

$$\sigma_m(z) = t_m + t_{m+1} - z, \;\; m = \overline{1,4} \;\; (t_5 = t_1)$$

transform D into a quadrangles which have a common side with it.

In paper [1] the equation

$$\sum_{m=1}^{4} (-1)^{m+1} f[\sigma_m(z)] = g(z), \;\; z \in D, \tag{1}$$

is studied under following conditions:

1) the term $g(z)$ is holomorphic in D and its boundary value $g^+(t) \in H(\Gamma)$;

2) we seek the solution in the class of functions $f(z)$ which are holomorphic outside D and vanish at infinity $\forall j \;\; f^-(t) \in H(l_j)$ and may have in vertices only logarithmic singularities. We will denote such class by B. A method equivalent to regularization was proposed for problem (1). We note that non-triviality of Eq. (1) follows from disconnectedness of the set $\mathbb{C} \setminus \cup\sigma_m(D)$, $m = \overline{1,4}$ (for more details see introduction in [2]). We establish in two particular cases the unconditional solvability of problem (1). Those cases are isosceles trapezoid D_1 with vertices $-2-i$, $2-$, $1+i$, $i-1$ and quadrangle D_2 with vertices $2^{-1}-i$, 2^{-1}, $2^{-1}+i$, -2^{-1}, one of angles is extended (see [2] and [1], respectively).

The goal of this paper is to investigate another equation

$$\sum_{m=1}^{4} f[\sigma_m(z)] = g(z), \;\; z \in D, \tag{2}$$

under same conditions and show that in particular cases of quadrangles D_1 and D_2 it has another picture of solvability. Let us note that Eq. (2) is more difficult than Eq. (1) (see Remark 1 below). The paper consists of three sections. In Item 1 we propose the method which is equivalent to regularization of Eq. (2). In Item 2 we show that in particular cases of quadrangles D_1 and D_2 it has the unique solvability

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