

Facially Symmetric Spaces and Predual Ones of Hermitian Part of von Neumann Algebras

M. M. Ibragimov^{1*}, K. K. Kudaibergenov^{1**}, and Zh. Kh. Seipullaev^{2***}

¹*Karakalpak State University named after Berdakh
ul. Akademika C. Abdirova 1, Nukus, 230113 Republic of Uzbekistan*

²*National University of Uzbekistan
ul. Durmon Yuli 29, Tashkent, 100125 Republic of Uzbekistan*

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Abstract—We prove that predual of real part of von Neumann algebra is strongly facially symmetric space if and only if is it a direct sum of Abelian algebra and algebra of I_2 type. At that, neutral strongly facially symmetric space is predual to Abelian algebra, only.

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INTRODUCTION

Study of the facially symmetric spaces, that appeared due to the geometric characterization of pre-conjugate spaces of JBW^* -triples admitting an algebraic structure, goes back to the papers of J. Friedman and B. Rousseau [1, 2]. Many of the properties required in these characterizations are natural assumptions for the state spaces of physical systems. Such spaces play the part of geometric models for states of quantum mechanics. Naturally, the predual space for complex von Neumann algebras and more general JBW^* -triples is a neutral strongly facially symmetric space [3].

Paper [4] gives a geometric characterization of complex Hilbert spaces and complex spin factors and a description of JBW^* -triples of rank 1 and 2 and Cartan factors of type 1 and 4. Later, J. Friedman and B. Rousseau in [5] described atomic facially symmetric spaces, showed that a neutral, strongly granular symmetric space is isometrically isomorphic to the predual space of one of the Cartan factors of type 1–6. M. Neil and B. Rousseau in [6] found geometric conditions under which the facially symmetric space is isometric to the predual space of a JBW^* -triple. In particular, it was proved that every neutral strongly facially symmetric space decomposes into a direct sum of atomic and non-atomic strongly facially symmetric spaces. Paper [7] gives a complete description of strongly facially symmetric spaces isometrically isomorphic to the predual space of the atomic commutative von Neumann algebra. In [8], the authors described strongly facially symmetric spaces isometrically isomorphic to the L_1 -space.

This paper is devoted to the study of spaces predual to real parts of von Neumann algebras. It is proved that the predual of the von Neumann algebra real part is a strongly facially symmetric space if and only if it is the direct sum of an Abelian algebra and an algebra of type I_2 . In this case, the neutral strongly facially symmetric spaces are necessarily predual to an Abelian algebra.

*E-mail: mukhtar_nukus@mail.ru.

**E-mail: karim2006@mail.ru.

***E-mail: jumabek81@mail.ru.