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Derivation of an Equation of Phenomenological Symmetry for Some Three-Dimensional Geometries

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Abstract—The main problems of the theory of phenomenologically symmetric (PS) geometries, i.e., geometries of maximum mobility, are their complete classification, the establishing of the fact of existence of their group symmetry, and finding an equation of the phenomenological symmetry for each of them. A complete classification of three-dimensional PS geometries has been already built. Their PS, i.e., the existence of a functional relation between the values of the metric function for all pairs of five points follows from the rank of the corresponding functional matrix. However, not for all such geometries an equation, which expresses the PS, is known in the explicit form. The paper describes methods of finding the equations of PS, which were applied to some three-dimensional geometries. For each of them we give groups of motions that define the group symmetry of degree six.

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INTRODUCTION

We consider three-dimensional geometries defined on a three-dimensional manifold $\mathfrak{M}$ by a (2-points) metric function $f : \mathfrak{M} \times \mathfrak{M} \to \mathbb{R}$, which relates a pair of points $\langle ij \rangle \in \mathfrak{M} \times \mathfrak{M}$ to a number $f(ij) \in \mathbb{R}$ ([1], § 4). Non-singularity of function f in its local coordinate representation

$$f(ij) = f(x_i, y_i, z_i, x_j, y_j, z_j) \tag{1}$$

is written in the form of the following two inequalities

$$\frac{\partial(f(ik), f(il), f(im))}{\partial(x_i, y_i, z_i)} \neq 0, \quad \frac{\partial(f(kj), f(lj), f(mj))}{\partial(x_j, y_j, z_j)} \neq 0 \tag{2}$$

for dense in $\mathfrak{M}^4$ sets of quadruples of points $\langle iklm \rangle$ and $\langle klmj \rangle$. We do not suppose the fulfillment of usual metric axioms for the function f, hence its value $f(ij)$ for the pair $\langle ij \rangle$ is not, generally, a distance between points i and j.

Phenomenological symmetry (PS) of rank 5 for three-dimensional geometry, defined by metric function (1), is expressed ([1], § 4) by the equation

$$\Phi(f(ij), f(ik), f(il), f(im), f(jk), f(jl), f(jm), f(kl), f(km), f(lm)) = 0, \tag{3}$$

valid for an open and dense in $\mathfrak{M}^5$ set of 5-tuple of points $\langle ijklm \rangle$. Note that, unlike the approach of L. Blumental [2], we do not fix function Φ in the left-hand side of Eq. (3) beforehand. We only suppose the existence of such function. It is known [3] that PS of rank 5 for a three-dimensional geometry is equivalent to its group symmetry of degree 6, which is defined by 6-parametric group of motions

$$x' = \lambda(x, y, z; a, b, c, p, q, r), \quad y' = \sigma(x, y, z; a, b, c, p, q, r), \quad z' = \tau(x, y, z; a, b, c, p, q, r), \tag{4}$$

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