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CRITERION OF ISOMORPHY OF NONCOMMUTATIVE ARENS ALGEBRAS

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1. *Introduction.* Let M be a semifinite von Neumann algebra, μ be exact normal semifinite trace on M, $(L^p(M,\mu), \|\ \|_p)$ be the Banach space of all μ-measurable operators adjoint to M and integrable with p-power. The set $L^\omega(M,\mu) = \bigcap\limits_{p\ge 1} L^p(M,\mu)$ is a filled $*$-subalgebra in the $*$-algebra $K(M,\mu)$ of all μ-measurable operators adjoint to M (see [1], [2]). Arens was first who considered algebras of that kind (see [3]) in the case where $M = L^\infty(0,1)$ and $\mu(f) = \int\limits_0^1 f\,dm$, where m is a linear Lebesgue measure. Later the properties of these algebras associated with an arbitrary commutative von Neumann algebra were studied in [4], [5]. For arbitrary von Neumann algebras, the M-algebras $L^\omega(M,\mu)$ were introduced in [1], [2] and were called the noncommutative Arens algebras.

In the present article we continue investigations of the properties of algebras $L^\omega(M,\mu)$. Here we obtain necessary and sufficient conditions for coincidence of the algebras $L^\omega(M,\mu)$ and $L^\omega(M,\nu)$, associated with different traces μ and ν. We establish the non–isomorphy of these algebras in the case where μ is a finite, ν is a semifinite but not finite traces on M. We find conditions upon the von Neumann algebras M and N and the traces μ and ν on M and N, respectively, which guarantee $*$-isomorphy of the Arens algebras $L^\omega(M,\mu)$ and $L^\omega(M,\nu)$.

All necessary notation and results from the theory of von Neumann algebras are taken from [6], and from the theory of noncommutative integration — from [7].

2. *Preliminary information.* Let M be a semifinite von Neumann algebra, μ be the exact normal semifinite trace on M, $K(M,\mu)$ be $*$-algebra of all μ-measurable operators adjoint to M (see [7]). We denote by $L^p(M,\mu)$ a Banach space of all $x \in K(M,\mu)$ such that $\|x\|_p = \mu(|x|^p)^{1/p} < \infty$, where $|x| = (x^*x)^{1/2}$ (see [8]). The set $L^\omega(M,\mu) = \bigcap\limits_{p\ge 1} L^p(M,\mu)$ is a filled linear subspace in $K(M,\mu)$. It was shown in [1], [2] that $L^\omega(M,\mu)$ is an $*$-subalgebra in $K(M,\mu)$ and with respect to the topology t_μ, generated by the system of norms $\{\|\ \|_p\}_{p\ge 1}$, $L^\omega(M,\mu)$ is a complete metrizable locally convex $*$-algebra.

Theorem 1. *Any $*$-isomorphism between the Arens algebras is a continuous mapping.*

We denote by $L_0(M,\mu)$ a linear subspace in $K(M,\mu)$, generated by the set $M \bigcup \Big(\bigcup\limits_{p>1} L^p(M,\mu)\Big)$. Each operator $y \in L_0(M,\mu)$ defines a linear continuous functional on $(L^\omega(M,\mu), t_\mu)$ via the formula $f_y(x) = \mu(xy)$. It was shown in [2] that any t_μ-continuous linear functional f on $L^\omega(M,\mu)$ can be represented in the form $f = f_y$ for a certain $y \in L_0(M,\mu)$, i. e., the space conjugate to the Arens algebra $(L^\omega(M,\mu), t_\mu)$ can be identified with the space $L_0(M,\mu)$.