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SOLVING OPERATOR AND INTEGRAL EQUATIONS BY THE BOGOLYUBOV–KRYLOV METHOD

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Introduction

During late 20s and early 30s of the XX century, for solving boundary integral equations, N.M. Krylov and N.N. Bogolyubov suggested (see, e. g., [1]) an original direct method. Later this method was widely used and remains be used for solving various classes of integral and integral-differential equations. A brief survey of the corresponding investigations and a series of obtained results can be found, for example, in [2]–[7].

In spite of the simplicity of the method and its ample practical applications, up to now its strict theoretical substantiation is absent. As for the proof of the convergence of the method, suggested by its authors, academicians N.M. Krylov and N.N. Bogolyubov for boundary integral equations, it is valid only under very rigid constraints upon the initial data.

In what follows a strict theoretical-functional substantiation of the Bogolyubov–Krylov method for operator equations of the second kind in normed spaces is suggested. In addition, we use essentially the general theory of approximation method by academician L.V. Kantorovich and the approximations by splines of minimal degrees. The general results obtained here are applied to various classes of regular and singular integral equations.

1. General operator scheme of the Bogolyubov–Krylov method and its substantiation

Let $M = M[a,b]$ and $C[a,b]$ be spaces of all bounded and continuous on segment $[a,b]$ functions, respectively, with the norms

$$\|\varphi\|_M = \sup_{a\le t\le b} |\varphi(t)|, \quad \varphi \in M; \quad \|f\|_C = \max_{a\le t\le b} |f(t)|, \quad f \in C.$$

On $[a,b]$ we introduce the node systems

$$t_k = a + k\frac{b-a}{n}, \qquad k = \overline{0,n}, \quad n \in \mathbb{N}, \tag{1.1}$$

$$\bar t_k = a + \left(k - \frac{1}{2}\right)\frac{b-a}{n}, \quad k = \overline{1,n}. \tag{1.2}$$

We denote by $\psi_k(t) = \psi_{k,n}(t)$, $k = \overline{1,n}$, the fundamental splines of the zero degree by the grid of nodes (1.1):

$$\begin{aligned} &\psi_k(t) = \{1 \text{ for } t \in [t_{k-1}, t_k); \;\; 0 \text{ for } t \notin [t_{k-1}, t_k)\}, \quad k = \overline{1, n-1}; \\ &\psi_n(t) = \{1 \text{ for } t \in [t_{n-1}, t_n]; \;\; 0 \text{ for } t \notin [t_{n-1}, t_n]\}. \end{aligned} \tag{1.3}$$