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A Criterion of Convergence of Lagrange–Sturm–Liouville Processes in Terms of One-Sided Module of Variation

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Abstract—We obtain a criterion of uniform convergence inside the interval $(0, \pi)$ of interpolation processes determined by eigenfunctions of the regular Sturm–Liouville problem with a continuous potential of bounded variation. The criterion is formulated in terms of one-sided modulus of variation.

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INTRODUCTION

G. I. Natanson in [1] obtained the Dini–Lipschiz criterion of uniform convergence inside the interval $(0, \pi)$, i.e., uniform on any compact subset of $(0, \pi)$ convergence of the Lagrange–Sturm–Liouville processes

$$L_n^{SL}(f, x) = \sum_{k=1}^{n} f(x_{k,n}) \frac{U_n(x)}{U_n'(x_{k,n})(x - x_{k,n})} = \sum_{k=1}^{n} f(x_{k,n}) l_{k,n}^{SL}(x), \tag{1}$$

where U_n is nth eigenfunction of a regular Sturm–Liouville problem

$$U'' + [\lambda - q]U = 0, \;\; U'(0) - hU(0) = 0, \;\; U'(\pi) + HU(\pi) = 0 \tag{2}$$

with continuous potential q of bounded variation on $[0, \pi]$, and with boundary-value conditions guaranteeing that the cosine is the main term of the asymptotic formulas for U_n, i.e., $h \neq \pm\infty$, $H \neq \pm\infty$. Here $0 < x_{1,n} < x_{2,n} < \cdots < x_{n,n} < \pi$ are roots of the function U_n. Approximative properties of the Lagrange–Sturm–Liouville operators (1) are studied also in works [2] and [3]. In paper [2] the author establishes the existence of a continuous on $[0, \pi]$ function such that its Lagrange–Sturm–Liouville interpolation process (1) unboundedly diverges a. e. on $[0, \pi]$. Papers [3–5] show that arbitrarily small variations of parameters of the Sturm–Liouville problem (2) (of the potential q or the constants h, H) lead to essential variation of the approximative properties of processes (1).

The properties of the Lagrange interpolation operators (1) are closely connected with behavior of sinc-approximations

$$L_n(f, x) = \sum_{k=0}^{n} \frac{\sin(nx - k\pi)}{nx - k\pi} f\left(\frac{k\pi}{n}\right) = \sum_{k=0}^{n} \frac{(-1)^k \sin nx}{nx - k\pi} f\left(\frac{k\pi}{n}\right), \tag{3}$$

used in the Whittecker–Kotel'nikov–Shannon countdown theorem [6]. The operators (3) are special case of the Lagrange–Sturm–Liouville operators (1) for boundary-value conditions of the first kind and null potential.

Analogous to the Lagrange–Sturm–Liouville operators (1) and sinc-approximations (3) interpolation operators have a great body of applications in numerical methods of mathematical physics,

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