ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 6, pp. 11–17. © Allerton Press, Inc., 2007.
Original Russian Text © V.P. Bulatov, T.I. Belykh, 2007, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2007, No. 6, pp. 14–20.

Numerical Solution Methods for Multiextremal Problems Connected with Inverse Problems in Mathematical Programming

V. P. Bulatov* **and T. I. Belykh**

Russian Academy of Sciences, Siberian Branch, L. A. Melent'ev Institute of Systems of Power Engineering, ul. Lermontova 130, Irkutsk, 664033 Russia

Received December 12, 2006

DOI: 10.3103/S1066369X07060023

Many mathematical problems are naturally associated with the family of the so-called inverse problems. Usually a pair of problems is called mutually inverse, if the definition of one of them contains a solution (or a part of a solution) to the other one. This approach is (in a sense) arbitrary, because it does not indicate which problem is direct and which one is inverse. The most investigated problem is more often considered as direct.

Let us adduce an example of a pair of such problems. Let a matrix A of dimensions $m \times n$ and vectors $c \in E^n$, $b \in E^m$ be given; it is required to find a vector

$$x^* \in \mathrm{Argmin}\{c^T x : x \in R\}, \tag{1}$$

$$R = \{x : Ax \le b,\ x \ge 0\}, \tag{2}$$

where R is a compact.

Vice versa: a vector x^* and a matrix A of dimensions $m \times n$ are given; it is required to find vectors $c^* \in R_c$, $b^* \in R_b$ such that

$$x^* = \underset{x}{\mathrm{argmin}}\{{c^*}^T x : Ax \le b^*,\ x \ge 0\}, \tag{3}$$

where R_c, R_b are convex compacts.

It is natural to treat problem (1), (2) as direct and problem (3) as the inverse linear programming problems. Evidently, problem (1), (2) corresponds to the whole family of inverse problems in form (3), depending on the choice of variable parameters of the technological matrix A, the price vector c, or the resource vector b.

1. INVERSE PROBLEMS IN MATHEMATICAL PROGRAMMING

In [1], a more general definition of the inverse problem in the mathematical programming is introduced; we use it below.

Let us have some parametric family of mathematical programming problems

$$\min_x\{\varphi(x,u) : g(x,u) \le 0,\ x \in R_x\}, \tag{4}$$

where $R_x \subset E^n$ is a compact, $u \in E^m$ is a vector parameter, $\varphi(x,u)$ is a continuous scalar function of its arguments, $g(x,u)$ is a continuous vector function ($g \in E^{m_1}$).

It is required to find a pair of vectors x^*, u^* from family (4) with the given properties

$$x^*, u^* \in R_{x,u} = \{x, u : f(x,u) \le 0,\ w(x,u) = 0,\ u \in R_u\}, \tag{4'}$$

where $f \in E^{m_2}$, $w \in E^{m_3}$ are continuous vector-functions, $R_u \subset E^m$ is a compact.

*E-mail: irk_elv@mail.ru.