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Construction of Polynomials of a Special Kind and Examples of Their Application in the Theory of Power Series and in the Wavelet Theory

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Abstract—In this paper we construct polynomials of a special type. We consider examples of their application for studying the convergence of special power series, for determining the upper and lower Riesz bounds for a basis consisting of B-splines, and for studying the convergence of a sequence of Battle–Lemarié scaling functions.

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1. INTRODUCTION

In many problems one needs analytical expressions for sums of functional series in the form $\sum\limits_{j=-\infty}^{\infty} \frac{1}{(x-j)^{2m}}$, $m = 1, 2, \dots$

Calculating exact Riesz bounds for a spline basis ([1], P. 150), Ch. Chui obtains the formula

$$\sum_{k=-\infty}^{\infty} \frac{1}{(x+\pi k)^{2m}} = -\frac{1}{(2m-1)!} \frac{d^{2m-1}}{dx^{2m-1}} \cot x.$$

Ch. Chui notes that this formula is exact and can be used as a tool for finding the Riesz bounds. However, in his opinion this formula is too complex for the application to splines of high orders m, that is why Ch. Chui uses other techniques for calculating the Riesz bounds.

In [2], P. 223, when obtaining the Battle–Lemarié wavelets, one proposes the following recurrent formulas for functions $p_n(\cos\xi) = \sin^{2n+2}\left(\frac{\xi}{2}\right) \sum\limits_{l} \frac{1}{(\xi/2+\pi l)^{2n+2}}$, $n = 1, 2, \dots$:

$$p_n(y) = \frac{1}{n(2n+1)}(-y(1-y)p'_{n-1}(y) - nyp_{n-1}(y) \\ + (1+y)((1-y)^2 p''_{n-1}(y) + 2n(1-y)p'_{n-1}(y) + n(n+1)p_{n-1}(y))),$$

where $'$ stands for the derivative in y, $y = \cos\xi$, and $p_0(y) \equiv 1$. By analyzing the formula we see that p_n are polynomials of degree n of the variable y. One can calculate them with the help of symbolic computing software.

In [3], P. 753, when finding the optimal quadrature formulas, we use the correlation

$$\sum_{j=-\infty}^{\infty} \frac{1}{(x-j)^2} = \frac{\pi^2}{\sin^2 \pi x}; \tag{1}$$

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