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On Riemann Boundary-Value Problem for Regular Functions in Clifford Algebras

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Abstract—We pose and investigate the Riemann boundary-value problem for regular and strongly regular functions in Clifford algebras. The posed problem is reduced to the matrix problem for analytical functions in one and two complex variables and we give its solution. We carry out the boundary-value problems in special cases.

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The monographs of N. I. Muskhelishvili ([1], pp. 416–446) and N. P. Vekua ([2], pp. 11–59) contain the linear conjugation problem for several unknown functions. The problem is to find a piecewise holomorphic vector $\Phi(z) = (\Phi_1, \Phi_2, \dots, \Phi_n)$ with a jump line L and having a finite order at infinity by the boundary condition

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in L, \tag{1}$$

where $G(t)$ is a matrix of class H defined on L and is not singular anywhere in L, and $g(t)$ is a vector of class H on L. We construct and study the canonical system of solutions to the homogeneous Riemann problem. We also give the solution to homogeneous and inhomogeneous Riemann problem (1), and obtain the conditions for the solvability of the inhomogeneous problem with a negative index.

V. A. Kakichev ([3], pp. 4–56) set and carried out an investigation of the Riemann boundary-value problem for analytic functions of two complex variables. The two-dimensional Riemann problem is to find the four functions $\Phi^{\pm\pm}(z, w)$ analytic in the domains $D^{\pm\pm} = D_1^{\pm} \times D_2^{\pm}$, respectively, by the boundary condition

$$A(t,\omega)\Phi^{++}(t,\omega) + B(t,\omega)\Phi^{-+}(t,\omega) + C(t,\omega)\Phi^{+-}(t,\omega) + D(t,\omega)\Phi^{--}(t,\omega) = F(t,\omega), \tag{2}$$

$$\Phi^{\pm=}(z_1,\infty) = 0, \ \ z_1 \in D_1^{\pm}, \ \ \Phi^{=\pm}(\infty, z_2) = 0, \ \ z_2 \in D_2^{\pm},$$

here the variables $(t,\omega) \in L^2 = L_1 \times L_2$, $L_1 = \partial D_1$, $L_2 = \partial D_2$, and the coefficients $A(t,\omega)$, $B(t,\omega)$, $C(t,\omega)$, $D(t,\omega)$, $F(t,\omega)$ are of class $H(L^2)$. In the general case, there is no solution to problem (2). If $A(t,\omega) = B(t,\omega), C(t,\omega) = D(t,\omega)$ or $A(t,\omega) = C(t,\omega)), B(t,\omega) = D(t,\omega)$, then we obtain the degenerate Riemann problems of the first kind that reduce to Riemann boundary-value problems for one variable with the second variable as a parameter.

The index $\chi_L(A)$ of a function $A(t,\omega)$ continuous on L^2 is defined ([3], pp. 10–11) as a change in its argument passing along $L : \chi_L = \frac{1}{2\pi i}\int\limits_L d\arg A(t,\omega)$. Assume the notation $\chi_1(A) = \chi_{L_1}(A)$ and $\chi_2 = \chi_{L_2}(A)$. It is proved in [4, 5] that if A, B, C, D, F are summable with degree $p > 1$ on the skeleton L^2 formed by simple closed Lyapunov curves, then conditions

1) $A \neq 0$, $B \neq 0$, $C \neq 0$, $D \neq 0$ on L^2,

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