Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 48, No. 1, pp.23–27, 2004 *UDC 517.977*

QUASISTABILITY OF VECTOR TRAJECTORY PROBLEM WITH THE PARAMETRIC OPTIMALITY PRINCIPLE

S.E. Bukhtoyarov and V.A. Yemelichev

As usual (see [1]–[5]), we understand the quasistability of a vector discrete optimization problem as a discrete analog of semicontinuity below (in Hausdorff's sense) of a multivalued mapping. The latter associates the set of problem parameters with the desired set of alternatives which are effective in a certain sense. So the quasistability of the problem is the property of preservation of the indicated set at "small" perturbations of the problem parameters. The quasistability radius is the limitary level of such perturbations.

In this paper, we consider the n-objective linear combinatory (on a system of subsets of some finite set) optimization problem whose optimality principle is defined with the help of the integer parameter s varying in the range from 1 to n. Besides, the extreme values of the parameter correspond to the Pareto and Slater optimality principles. For each value of the parameter s we obtain the formula for the quasistability radius of the problem and formulate the necessary and sufficient conditions of stability of this type.

The class of vector (n-objective) problems which are considered in this paper can be described by the following combinatory model. We assume that given is the vector criterion

$$f(t,A) = (f_1(t,A_1), f_2(t,A_2), \dots, f_n(t,A_n)) \to \min_{t\in T}$$

with the partial criteria

$$f_i(t,A_i) = \sum_{j\in N(t)} a_{ij}, \quad i \in N_n,$$

where T is the set of trajectories, $T \subseteq 2^E$, $|T| \geq 2$, $E = \{e_1, e_2, \dots, e_m\}$, $m \geq 2$, $N_n = \{1, 2, \dots, n\}$, $n \geq 1$, $N(t) = \{j \in N_n : e_j \in t\}$, A_i is the i-th row of the matrix $A = [a_{ij}] \in \mathbf{R}^{n\times m}$. We assume that $f_i(\emptyset, A_i) = 0$.

Using the binary relation which depends on the integer parameter we introduce n optimality principles (in other terms, the choice functions). For that we define on the set of trajectories T for each index $s \in N_n$ the binary relation of strict preference $\Omega_s^n(A)$ of the trajectories by the formula

$$t\Omega_s^n(A)t' \Leftrightarrow [t,t',A]^+ > (s-1)[t,t',A]^0 + (n-1)[t,t',A]^-,$$

where

$$[t,t',A]^+ = |\{i \in N_n : g_i(t,t',A_i) > 0\}|,$$
$$[t,t',A]^0 = |\{i \in N_n : g_i(t,t',A_i) = 0\}|,$$
$$[t,t',A]^- = |\{i \in N_n : g_i(t,t',A_i) < 0\}|,$$
$$g_i(t,t',A_i) = f_i(t,A_i) - f_i(t',A_i).$$

This work was supported by the Byelorussian State University as a part of the State Basic Research Program of Byelorussia "Mathematical structures 29".

©2004 by Allerton Press, Inc.
Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of $50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.