ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 5, pp. 23–33. © Allerton Press, Inc., 2012.
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Method of Semidefinite Lyapunov Functions for Systems of Nonautonomous Differential Equations

B. S. Kalitin[1*] and R. Chabour[2]**

[1] *Belarusian State University, pr. Nezavisimosti 4, Minsk, 220030 Republic of Belarus*
[2] *Metz University P. Verlin, Il du Saulcy, 57045, Metz-Cedex 01, France*

Received January 27, 2011

Abstract—We extend the method of semidefinite Lyapunov functions for analyzing the motion stability as applied to systems of nonautonomous differential equations. We prove basic stability theorems and illustrate them with examples.

DOI: 10.3103/S1066369X12050039

Keywords and phrases: *nonautonomous systems of differential equations, equilibrium, stability, Lyapunov function.*

The method of Lyapunov functions ([1], pp. 77–94) is used in many problems of control and automatic regulation of dynamic processes. For nonautonomous differential equations it was developed, e.g., in papers [2–6] by N. G. Chetaev, K. P. Persidskii, J. Kurzweil, E. A. Barbashin, N. N. Krasovskii, V. I. Zubov, V. M. Matrosov, S. N. Vasil'ev, J.-P. La Salle, T. Yoshizawa, Z. Artstein, and A. S. Andreev. Here in problems of stability, asymptotic stability, and stability in whole, essential is the requirement that the auxiliary function $V(x,t)$ should be positive definite. In papers [6–8] for the asymptotic stability problem the requirements to the time derivative of the Lyapunov function $\dot{V}(x,t)$ are weakened. We prove that instead of a negative definite function $\dot{V}(x,t)$ one can use a Lyapunov function with a semidefinite derivative whose zero set contains no positive half-trajectories.

In papers [9–19] one develops the second method for nonautonomous systems in the case when the Lyapunov function $V(x,t)$ is only positive definite. The specific feature of these stability results consists in the fact that the Lyapunov function can vanish not only at the origin of coordinates of the phase space, but also on the zero level surface containing the origin. Naturally, the time derivative of such a function is only negative definite. The statements of the corresponding results differ from each other in accordance with three approaches to studying the stability of the zero solution.

The first of them [9, 12–15] is based on the technique of limit equations [5], where together with the notion of limit functions of right-hand sides of the system (the limit equations) one also uses the notion of Lyapunov limit functions.

The second approach is based on the separation of variables (coordinates of the system state) onto subspaces $x = (y, z)$ with an explicit representation of the manifold $(y = \varphi(t, z))$, where the Lyapunov function equals zero [10, 11, 18].

The third approach [17–20] is based on methods of the qualitative theory of differential equations, namely, on studying the local and global properties of half-trajectories in a neighborhood, where the Lyapunov function vanishes. Here we use the notions of the "threshold property" [21], the uniform integral continuity, properties of limit sets of solutions [21–23], etc.

In this paper we use an approach close to the third one mentioned above and generalize results on the stability of the zero solution obtained in [17, 19].

*E-mail: Kalitine@yandex.ru.
**E-mail: rachid.chabour@univ-metz.fr.