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INFINITESIMAL ARG-DEFORMATIONS OF SURFACES UNDER THE CONDITION OF GENERALIZED SLIDING

V.V. Sidoryakina

We study $(m+1)$-connected, $m \geq 0$, surfaces F of positive Gaussian curvature with boundary ∂F in the three-dimensional Euclidean space E^3. We suppose that F is convex downwards with respect to the rectangular coordinate system $Oxyz$ under consideration, projects bijectively onto a domain D of the coordinate plane Oxy and is given by an equation $z = f(x, y)$, $(x, y) \in D$, $f \in C^{3,\alpha}(\overline{D})$, $0 < \alpha < 1$, $\overline{D} = D + \partial D$, where ∂D is the boundary of D, $\partial D \in C^{2,\alpha}$. Under the above assumptions, we will say that F satisfies the regularity conditions.

Consider infinitesimal ARG-deformations of a surface F: $\overline{r} = \overline{r}(x, y)$, $(x, y) \in D$, with given recurrence coefficient λ characterized by the following properties:

1) spherical (Gauss) image of F is pointwise stationary, i. e., the variation $\delta\overline{n}$ of the unit normal vector $\overline{n}$ of F equals zero;

2) the variation $\delta(d\sigma)$ of the area element $d\sigma$ satisfies, at any point of F, the equation

$$\delta(d\sigma) = 2\lambda H(\overline{U}, \overline{n})d\sigma + g d\sigma, \tag{1}$$

where $\overline{U}$ is the field of displacements of the points of F under the deformation, H the mean curvature of F, g a given function of class $C^{2,\alpha}$, $0 < \alpha < 1$, and λ is a given number.

It is known [1] that ARG-deformations are described by the equation

$$[\overline{r}_y, \overline{U}_x] + (2\lambda H(\overline{U}, \overline{n}) + g)[\overline{r}_x, \overline{r}_y] = [\overline{r}_x, \overline{U}_y] \quad \text{on} \quad F. \tag{2}$$

According to [2], under the above conditions, for $\lambda = 0$, $g = 0$ in (1), the arbitrariness of existence of infinitesimal ARG-deformations of F is very high. The aim of this paper is to single out a class of external constraints imposed on the behavior of the surface under deformation under which the surface admits a unique infinitesimal deformation (or a set of infinitesimal deformations depending on a finite number of parameters).

We consider the exterior constraint which is the condition of generalized sliding of the following form

$$(\overline{U}, \overline{l}) = h \quad \text{on} \quad \partial F, \tag{3}$$

where $\overline{l}$ is a given vector field, $|\overline{l}| \neq 0$, and h is a function of class $C^{1,\alpha}$, $0 < \alpha < 1$, defined on ∂F.

We describe the external constraints with respect to the behavior of F under an ARG-deformation by considering a family of external constraints with given vector field of the form $\overline{l} = \overline{\nu} + \gamma\overline{n}$, $\overline{l} \in C^{1,\alpha}$, $0 < \alpha < 1$, where $\overline{\nu}$ is the unit vector of the exterior normal of D in the plane Oxy and γ is a given function of class $C^{1,\alpha}$, $0 < \alpha < 1$.

As in [3], an external constraint (3) will be said to be well-posed with respect to infinitesimal ARG-deformations with given recurrence coefficient λ in the regularity class $C^{1,\alpha}$ if the surface F admits a unique infinitesimal ARG-deformation with field of displacements $\overline{U}$ compatible with this