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On the Approximate Conformal Mapping of the Unit Disk on a Simply Connected Domain

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Abstract—We present a method of constructing the analytic function realizing an approximate conformal mapping of the unit disk on an arbitrary simply connected domain with the given smooth parametrically defined boundary. The method is based on a new boundary parameterization. Solution of the problem is reduced to the Fredholm integral equation of the second kind. We present the examples of three ways to solve the integral equation.

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INTRODUCTION

This paper is devoted to construction of the approximate conformal mapping of the unit disc onto any simply connected domain with the smooth boundary. In Conclusion we discuss the case of the boundary with corners. There exist many approximate methods of conformal mapping construction. Some of them are based on integral equations [1–5]. In recent years due to appearance of high-performance computers numerical methods turned out to be actual. For example, the method presented in [6] is based on the arbitrary domain circular packing, which seems to be rather complicated task. Analytic methods clearly have certain advantages over numerical ones in the following matters: We can obtain sufficiently smooth solution and apply the differential calculus to the solution. Particularly, the polynomial representation of the solution makes it possible to find the analytical solutions of the basic elasticity theory for the plane domains. Here we present the method which is an analog of one introduced by Theodorsen–Garrick [1] but differs from it by the new parametric representation of the boundary of the domain. The solution to the problem is reduced to the Fredholm integral equation of the second kind. We give three solution methods illustrated by construction of three different mapping. Particularly, we give the Taylor series expansion of the conformal mapping which transforms the unit disk to the domain arbitrary close to elliptic one.

REDUCTION OF THE PROBLEM TO THE INTEGRAL EQUATION

Let the boundary of the simply connected domain D be the simple smooth closed curve L with its parametric representation

$$\{x = x(t),\ y = y(t),\ t \in [0, 2\pi]\},$$

here the functions $x(t)$ and $y(t)$ are 2π-periodic. So we represent these functions as Fourier series. Therefore the complex representation of the boundary L has the form

$$z(t) = x(t) + iy(t) = \sum_{k=-\infty}^{+\infty} c_k e^{ikt}.$$

We assume that the boundary is traversed in the positive direction of the parameter t, otherwise we change the sign of t.

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