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Attraction Sets in Abstract Attainability Problems: Equivalent Representations and Basic Properties

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Abstract—We consider attainability problems in topological spaces under asymptotic constraints.

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1. INTRODUCTION

In this paper we use the following abbreviations: FB (filter base), MS (measurable space), AS (attraction set), DS (directed set), AD (attainability domain), GE (generalized element), s/s (subset), ApS (approximate solution), TS (topological space), and u/f (ultrafilter).

We consider general questions connected with the asymptotic attainability; the latter is understood as the attainability of certain states under asymptotic constraints. One natural concrete variant of the problem which we consider below is related to the construction of the AD of a controlled system at a fixed time moment and the study of its properties. This concrete problem is not stable with respect to disturbances, in particular, for weakened constraints. One can treat weakened constraints (phase constraints, edge and intermediate ones) in this problem as asymptotic restrictions. Respectively, the limit of real AD which correspond to weakened constraints can be considered as an AS; from the practical point of view it plays the same role as a "regular " AD does. Together with AD, with the strict and approximate fulfillment of constraints one can consider sheaves of trajectories of the controlled system and thus obtain sets in a functional space equipped with the corresponding topology. The latter can be generated by a metric, but this is not necessarily so (for example, in cases typical for control problems with continuous time, the topology of pointwise convergence is not metrizable). Therefore it is natural to consider logically similar problems on the attainability in TS, though this can require the application of more complex constructions for AS. This approach is used in the present paper.

In addition, using such general statements of the problem, it is natural to assume asymptotic analogs of constraints which are not necessary connected with the weakening of some standard conditions. Really, weakening conditions of such a kind, we form a family of sets of feasible elements for each constraint weakened in a concrete way (generated by the set of mentioned conditions). From this moment we "deal" with the obtained family (really, this is a result of the relaxation procedure for the initial problem), using the minimal set of properties of sets that compose it. But this means that from the beginning we can assume that the family of s/s of the space of regular solutions (or controls) is given, not questioning how this family has been obtained (it is advisable to reduce the set of properties "to the minimum"). In particular, we do not tend to connect this family with a successive weakening of some "rigid " conditions. In this case the question on the attainability in TS has an asymptotic character.

Let us consider a very simple example of a static problem, fixing two finite-dimensional spaces $\mathbb{R}^m$ and $\mathbb{R}^n$ (m and n are natural numbers), and a bounded mapping f from $\mathbb{R}^m$ to $\mathbb{R}^n$. For definiteness,

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