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A QUASIREGULAR ASYMPTOTIC OF A SOLUTION OF THE SINGULARLY PERTURBED CAUCHY PROBLEM FOR LINEAR SYSTEMS OF DIFFERENTIAL MATRIX EQUATIONS

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We propose a method different from the known ones for constructing a quasiregular asymptotic of a solution of singularly perturbed initial problems for linear systems of differential matrix equations which arise in the study of some applied problems. We assume that the solution singularities which reflect the structure of the boundary layer are written in the closed analytic form, and other components of the asymptotic depend regularly on a small parameter.

Theorem 1. *The singularly perturbed Cauchy problem in* R^n

$$\varepsilon \dot{Z} = A(t)Z + ZB(t), \quad Z(0,\varepsilon) = Z^0, \tag{1}$$

($A(t), B(t) \in C^\infty[0,1]$, $A(t)$, $B(t)$, Z *are* $n\times n$*-matrices) when the spectra* $\{\lambda_{Aj}(t)\}_1^n$ *and* $\{\lambda_{Bj}(t)\}_1^n$ *of the matrices* $A(t)$ *and* $B(t)$ *fulfill the conditions*

$$\begin{aligned} \sigma_{Ajk}(t) &\equiv \lambda_{Aj}(t) - \lambda_{Ak}(t) \neq 0, \quad \operatorname{Re}\lambda_{Aj}(t) \leq 0, \\ \sigma_{Bjk}(t) &\equiv \lambda_{Bj}(t) - \lambda_{Bk}(t) \neq 0, \quad \operatorname{Re}\lambda_{Bj}(t) \leq 0 \end{aligned} \tag{2}$$

($j \neq k$, $j,k = \overline{1,n}$, $t \in [0,1]$), *has a unique and uniformly bounded on the segment* $[0,1]$ *solution for* $\varepsilon \to +0$. *This solution is representable in the quasiregular form*

$$\begin{aligned} Z(t,\varepsilon) &= S_A(t)\Big(E + \sum_{k=1}^N \overline{\overline{H}}_{Ak}(t)\varepsilon^k\Big) \exp\Big(\varepsilon^{-1}\int_0^t \sum_{k=0}^N \Lambda_{Ak}(s)\varepsilon^k ds\Big) Z^0 \times \\ &\times \exp\Big(\varepsilon^{-1}\int_0^t \sum_{k=0}^N \Lambda_{Bk}(s)\varepsilon^k ds\Big)\Big(E + \sum_{k=1}^N \overline{\overline{H}}_{Bk}(t)\varepsilon^k\Big) S_B^T(t) + O(\varepsilon^{N+1}) = S_A(t)\Big(\sum_{k=0}^N P_{Ak}(t)\varepsilon^k\Big) \times \\ &\times \exp\Big(\varepsilon^{-1}\int_0^t \Lambda_{A0}(s)ds\Big) Z^0 \exp\Big(\varepsilon^{-1}\int_0^t \Lambda_{B0}(s)ds\Big)\Big(\sum_{k=0}^N P_{Bk}(t)\varepsilon^k\Big) S_B^T(t) + O(\varepsilon^{N+1}), \end{aligned} \tag{3}$$

$$\begin{aligned} S_A^{-1}(t)A(t)S_A(t) &= \Lambda_{A0}(t) = \operatorname{diag}\{\lambda_{A10}(t),\dots,\lambda_{An0}(t)\}, \\ S_B^{-1}(t)B^T(t)S_B(t) &= \Lambda_{B0}(t) = \operatorname{diag}\{\lambda_{B10}(t),\dots,\lambda_{Bn0}(t)\}, \end{aligned}$$

where the matrix functions $\overline{\overline{H}}_{Ak}(t)$, $\overline{\overline{H}}_{Bk}(t)$, $\Lambda_{Ak}(t)$, $\Lambda_{Bk}(t)$ *are defined uniquely during the proof.*

Proof. The solution of problem (1), taking into account [1], is representable in the form

$$Z(t,\varepsilon) = X(t,\varepsilon)Y(t,\varepsilon), \tag{4}$$